SCIENTIFIC OASIS

Decision Making Advances

Journal homepage: www.dma-journal.org
ISSN: 2956-2384

# Scrutiny of Neutrosophic Cubic Fuzzy Data with Different Parametric Values using Bonferroni Weighted Mean: A Robust MCDM Approach

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**ARTICLE INFO**

***Article history:***
Received 29 December 2024
Received in revised form 1 March 2025
Accepted 3 March 2025
Available online 3 March 2025

***Keywords:***
Neutrosophic Cubic Fuzzy Sets; Bonferroni Mean; Aggregating Operator; Multiple-Criteria Decision-Making.

**ABSTRACT**

Neutrosophic cubic interval-valued fuzzy data-based aggregation operators are critical tools for addressing the intricacy of modern decision-making problems. They can handle diverse forms of fuzziness, uncertainty, and indeterminacy, which makes them able to tackle problems like product selection, resource allocation, and project prioritization. Taking the full support of Bonferroni mean, the current approach focuses on two novel aggregation operators such as Neutrosophic cubic fuzzy arithmetic Bonferroni mean $NCFABM^{(x,y)}$ and weighted Neutrosophic cubic fuzzy arithmetic Bonferroni mean $WNCFABM^{(x,y)}$. In addition, parametric value is also incorporated, which could lead to presenting a summative assessment in diverse scenarios. Furthermore, proposed aggregation operators are employed in multiple-criteria decision-making environments to solve numerical examples based on real-life problems; i.e. usefulness of the suggested aggregation operators in the system for managing health care. Finally, in support of our provided work, a detailed comparison analysis based on the suggested scheme versus current schemes based on self-defeating scenarios has been added to demonstrate reliability and validity.

## 1. Introduction

It might be difficult to distinguish between legal and invalid answers in decision-making, as well as between logical and illogical ones when there are several attributes and objectives involved in the issue. In these circumstances, decision-makers become increasingly bewildered and reluctant. Zadeh [1] invented fuzzy sets (FSs) to cope with this sort of information, which has sparked a lot of attention. After the fruitful utilization of the theory of FSs, numerous other scientists and educators in a variety of fields have expanded upon the notion. Because of the expeditiously growing complexity of multi-value data about the socioeconomic environment or scarcity of knowledge regarding multiple-criteria decision-making (MCDM) challenges, real-world decision-making problems present

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https://doi.org/10.31181/dma31202576

significant challenges over time because decision information is frequently insufficient to identify the information. As a result, it is highly challenging for the decision-makers to give an accurate answer because information is never certain, clear, or specific to handle these kinds of common data. Information ambiguity is another problem. Like the previous sort of uncertainty, when the researchers are unable to distinguish between two distinct items, intuitionistic FSs [2] are employed to cope with vagueness.

In order to handle doubts and ambiguities, Molodtsov [3] finally offered soft sets (SSs). This study has been productively employed in a variety of applications comprising cognitive science, activity knowledge, and probability [4]. A logical analysis of the soft sets by Maji *et al*. [5, 6] is presented, and it includes all necessary operators and properties. Fuzzy soft sets [7] and intuitionistic fuzzy soft sets [8] were added to the research to address uncertainty and ambiguity.

The neutrosophic set (NS), a phrase that is far superior for representing that kind of information, was thus proposed by Smarandache [9, 10]. By extending both the classical and fuzzy analogies, neutrosophy serves as the grounds for an entire assemblage of innovative mathematical theories. Truth ($T$), indeterminacy ($I$), and false ($F$) membership grades, all of which are defined over an underlying set $X$, are three related defining functions for an element in a neutrosophic set. These three tasks are wholly separate from one another. The neutrosophic set examines how neutralities interact with ideational spectra as well as their origins, traits, and range. The typical FS concept is expanded upon by the notion of NS [11], and so on [12-13]. Numerous areas have extensive uses for the neutrosophic set [14-16]. From a philosophical standpoint, the truthness, indecision, and untruth of an NS are three invariants falling in [0, 1]. Maji [17] has broadened the definition of SS to include neutrosophy by inaugurating the study of neutrosophic soft sets (NSSs). Deli and Broumi [18] provided this concept's matrix representation and aggregation operators. Numerous mathematicians have developed TOPSIS, VIKOR, and other algorithms for their examination work in several scientific disciplines intending to solve issues regarding MCDM [19, 20]. Additionally, this knowledge is employed to develop further theories of decision-making and is implemented in the neutrosophic environment [21–23].

By integrating the SS and its composites with the fuzzy, intuitionistic, and neutrosophic hyper soft sets, Smarandache [24] presented a different strategy for handling ambiguity. By making the function a multi-argument function, he was able to do this. Next, Wang *et al*. [25, 26] developed the conceptions of interval neutrosophic set (INS) as well as single-valued neutrosophic set (SVNS), which are variants of NS. These concepts specified set-theoretic operators and several additional features of SVNSs and INSs. Due to their limited efficiency in scenarios involving collective decision-making, conventional FSs were extremely difficult for decision-makers to deal with. Meanwhile, Zhang *et al.* [27] investigated an outranking approach for DM under the INS environment. In order to indicate the degree of confidence or membership, Jun *et al.* [28] introduced the concept of cubic sets (CSs), which use both interval-valued and single-valued FSs. Additionally, as time went on, several academics investigated some novel extensions in the form of CSs with their extensive forms. Zhan *et al.* [29] and Jun *et al*. [30] explored neutrosophic cubic fuzzy sets (NCFSs) with several key properties by effectively applying cubic FSs to NSs. Later, under a broad spectrum of elegant theories, a hybridized form of NCFS turned out to be swiftly created for use in many applications. Neutrosophic cubic sets offer more reliable information than their generic counterparts. As a result, neutrosophic cubic fuzzy sets offer a more effective and superior solution to problems involving many criteria in group decision-making.

The most interesting and important aspect of contemporary group decision theory is its concerns with MCDM. Before picking the finest choice, multiple influential attributes are mandatory for the selection of a set of alternatives. An essential topic is how to communicate preference values in decision-making situations to obtain solutions to these hard issues. Preference relations (PRs) may be utilized without much effort in several group decision-making problems. While working with these types of issues (group decision-making problems), decision experts should make pairwise judgments of alternatives to show their preference data for alternative evaluation concerning chosen attributes' values. One of the crucial steps in resolving issues with MCDM and other utilities and selecting the most appropriate alternative is aggregation, which calls for the opinions of the decision experts to be combined using an adequate aggregation operator. Operators of aggregations are crucial to decision-making. Aggregation operators primarily serve as a tool to combine the estimates of multiple decision-makers into a single estimate. Over the past three decades, a variety of aggregation operators have emerged in the field of decision-making. These profound operators are extensively examined in different classical and non-classical decision theories. These well-found operators have a variety of applications in multiple fields like artificial intelligence, economics, mathematical modeling, robotics, biology, economics, etc. The optimum approach to handling data in each of these situations was to use a range of aggregation operators. However, highly uncommon operators are accessible for neutrosophic cubic environments [31, 32].

Neutrosophic cubic sets depict real decision-making difficulties better owing to the reason that they can handle a greater amount of unreliable, inconsistent, and uncertain information [33]. In addition, Ajay *et al*. [34] studied the neutrosophic cubic fuzzy Bonferroni geometric operator. The study on Einstein's geometric aggregation operators under a neutrosophic cubic environment was coined by Khan *et al*. [35]. The research of Zhan *et al*. [36] was focused on MCDM on neutrosophic cubic sets. Neutrosophic cubic sets were subjected to grey rational analysis (GRA) methods by Banerjee *et al*. [37]. Cosine measure was established for a neutrosophic cubic set by Lu and Ye [38]. Pramanik *et al*. [39] successfully employed similarity measures to neutrosophic cubic sets. Under the neutrosophic cubic environment, Dombi aggregation operations were explored by Shi and Ji [40]. Aggregation operations over neutrosophic cubic numbers were defined by Ye [41]. A hybrid geometric aggregation operator for neutrosophic cubic collections accompanied by some of its peculiar attributes was presented and practically implemented by Alhazaymeh *et al*. [42].

The purpose of the literature review is to familiarize the readers with the ideas behind the Bonferroni aggregation technique when used with neutrosophic fuzzy data in multi-attribute group decision-making (MAGDM) techniques. Due to the way it is constructed, which divides it into two parts, the Bonferroni mean [43] aggregation operator is unique. The product of one argument and the mean of remaining inside arguments equals each argument of the outer arithmetic mean. In the past, both the arithmetic and generalized means have been applied in similarity classifiers [44]. Ajay *et al*. [45] employed a geometric Bonferroni mean operator in neutrosophic cubic fuzzy settings following the conceptions of neutrosophic cubic sets [46]. Recently, an application in business analytics was presented by Darvesh *et al*. [47]. Bonferroni means in intuitionistic fuzzy environment have been discussed in [48]. Under an interval-valued fuzzy environment, optimistic as well as pessimistic decision-making with dissonance reduction has been studied in [49]. Hamidi and Rasuli [50] presented the notion of normed bifuzzy valued ideals of semi-groups. A comprehensive study on Q^*-closed sets in fuzzy neutrosophic topology was performed by Mohammed *et al*. [51]. Sing and Sharma [52] unveiled divergence measures and aggregation operators for SVNSs.

After a careful review of the literature, it is found that aggregation based on parametric induction for single-valued and multiple-valued neutrosophic cubic fuzzy cubic information needs to be uncovered. Therefore, the comprehensive aim of this study is to extend the weighted Bonferroni arithmetic operator for interval-valued NCSs for different cases of parameter values. For this purpose, we first presented the weighted neutrosophic cubic fuzzy Bonferroni arithmetic mean (WNCFBAM) aggregation operator accompanied by its prime characteristics. Secondly, we developed the MCDM methodology predicated on the suggested operator (i.e. WNCFBAM) to categorize the options for neutrosophic cubic fuzzy data.

### *1.1 Motive of the Study*

The main motive behind conducting this study arises from the need for enhanced decision-making frameworks capable of handling complexity and high uncertainty in big data, where conventional procedures often fail. Founded upon Neutrosophic cubic fuzzy sets, the aggregating operators provide a dual-layer approach to capture these uncertainties more comprehensively compared to conventional approaches, which makes them highly relevant for applications in fields such as risk assessment, financial problems, and supply chain management. In addition, neutrosophic cubic fuzzy-based Bonferroni weighted mean significantly enhances MCDM processes, because of its flexible and nuanced data aggregation tool. The study primarily focuses on the practical challenges in management and operations research by advancing the theoretical foundation of Neutrosophic cubic fuzzy sets. Furthermore, creating tools that can better capture the complexities of real-world data, contributes to the growing field of neutrosophic and fuzzy set theory.

### *1.2 Application*

The current research is very significant in many applications, especially in operations research and management sciences. It primarily emphasizes decision-making processes that require handling complex, imprecise, or uncertain data. Some of the major applications are highlighted below.

i. *Resource allocation and optimization* – Allocating resources optimally under uncertain conditions is crucial in operations research. Traditional methods might struggle with ambiguity in data. However, NCFSs in conjunction with Bonferroni weighted mean enable a more refined aggregation that takes parametric variations into account, and it can support us in resource allocation tasks such as supply chain management or scheduling, where decisions need to account for both quantitative and qualitative factors that are not entirely certain. For example, managing inventory levels while considering fluctuations in demand and supply conditions can be optimized using this approach.

ii. *Strategic planning and risk management* – Businesses face risks that are hard to quantify, such as customer satisfaction or market volatility. In such situations, fuzzy systems based on neutrosophic cubic fuzzy sets are more volatile because they can handle uncertainty better than traditional probabilistic methods. In Strategic planning, this method can help organizations develop more robust plans by considering a range of probable outcomes under different parametric conditions. This enables us to assess scenarios where risk factors are uncertain but crucial, like market entry strategies or investment portfolio optimization.

iii. *Enhanced decision-making* – Decision-making often involves uncertainty and multiple criteria due to incomplete or imprecise data, which are crucial in management and operations. In these scenarios, this approach could improve the whole decision-making process, because it allows for a superior representation of this incertitude by capturing both memberships (truth and false) simultaneously. This approach could improve decision-making processes in risk assessment, project prioritization, and supplier selection by allowing managers to weigh complex, fuzzy criteria and make more informed decisions with a structured approach to uncertainty.

The body of this work is systematized along these lines: the second and third sections consist primarily of certain essential principles and helpful operational rules of neutrosophic cubic fuzzy sets. The next two sections discuss the notion of the Bonferroni means aggregation operator, WNCFBAM, and some impressive hypothetical findings. To verify the viability, we gave an example based on the suggested model in Section 6. In closing, we wind up the work with a look ahead in Section 7.

## 2. Preliminaries

This segment is devoted to some basic concepts to better comprehend the rest of the article. In addition, some common operational rules are also added.

**Definition 1.** An NCFS *S* is over *X* can be specified by focusing [30] as:

$$S=\{\chi,(T(\chi),I(\chi),F(\chi));(T_\omega(\chi),I_\omega(\chi),F_\omega(\chi))\,|\,\chi\in X\}. \tag{1}$$

In this expression, the terms $T(\chi)$, $I(\chi)$, and $F(\chi)$ denote the interval-valued neutrosophic sets with truth, indeterminacy, and false interval values as given below:

$$\left.\begin{array}{l} T(\chi)=[T_{\min}(\chi),T_{\max}(\chi)]\subseteq[0,1] \\ I(\chi)=[I_{\min}(\chi),I_{\max}(\chi)]\subseteq[0,1] \\ F(\chi)=[F_{\min}(\chi),F_{\max}(\chi)]\subseteq[0,1] \end{array}\right\}. \tag{2}$$

The corresponding membership values associated with these grades are $\langle T_\omega(\chi),I_\omega(\chi),F_\omega(\chi)\rangle\in[0,1]$. Simply, an NCFS is designated by the expression $S=\{\chi,\langle T,I,F\rangle;\langle T_\omega,I_\omega,F_\omega\rangle \mid \chi\in X\}$, where each component $\langle T,I,F\rangle\subseteq[0,1]$ in $\langle T_\omega,I_\omega,F_\omega\rangle\in[0,1]$ must fulfill the below criteria:

$$\left.\begin{array}{l} 0\le T_{\max}+I_{\max}+F_{\max}\le 3 \\ 0\le T_\omega+I_\omega+F_\omega\le 3 \end{array}\right\} \tag{3}$$

**Definition 2.** Suppose $S$ be an NCFS in a universal set *X*. According to [46], the external and internal NCFs in *X* can be outlined as:

$$S=\left\{\begin{array}{l}[T_{\min}(\chi),T_{\max}(\chi)],[I_{\min}(\chi),I_{\max}(\chi)],[F_{\min}(\chi),F_{\max}(\chi)] \\ \text{with } \langle T_\omega(\chi),I_\omega(\chi),F_\omega(\chi)\rangle \mid \chi\in X\}\end{array}\right\} \tag{4}$$

$S$ is said to be internal/external if for all permissible values of $\chi$ when:

$$\left.\begin{array}{l} T_{\min}(\chi)\le T_\omega(\chi)\le T_{\max}(\chi) \\ I_{\min}(\chi)\le I_\omega(\chi)\le I_{\max}(\chi) \\ F_{\min}(\chi)\le F_\omega(\chi)\le F_{\max}(\chi) \end{array}\right\} \tag{5}$$

$$\left.\begin{array}{l} T_\omega(\chi)\notin\left[T_{\min}(\chi),T_{\max}(\chi)\right] \\ I_\omega(\chi)\notin\left[I_{\min}(\chi),I_{\max}(\chi)\right] \\ F_\omega(\chi)\notin\left[F_{\min}(\chi),F_{\max}(\chi)\right] \end{array}\right\} \tag{6}$$

**Definition 3.** Suppose $S$ is an NCFS in $X$, then the support can be defined by focusing [45] as:

$$\left\{\begin{array}{c}\left[T_{\min}(\chi),T_{\max}(\chi)\right]\supset[0,0],\ \left[I_{\min}(\chi),I_{\max}(\chi)\right]\supset[0,0],\ \left[F_{\min}(\chi),F_{\max}(\chi)\right]\subset[1,1] \\ \left\langle T_\omega(\chi)\succ 0, I_\omega(\chi)\succ 0 \text{ and } F_\omega(\chi)\prec 1\right\rangle \mid \chi\in X\end{array}\right\} \tag{7}$$

**Definition 4.** Let $s\neq\varphi$ denote an NCFS in $X$. The score, accuracy, and certainty functions are defined as per earlier reported study in [41] as follows:

$$\hat{s}(s)=\frac{1}{2}\left(\frac{4+T_{\max}(\chi)-I_{\max}(\chi)-F_{\max}(\chi)+T_{\min}(\chi)-I_{\min}(\chi)-F_{\min}(\chi)}{6}+\frac{2+T_\omega(\chi)-I_\omega(\chi)-F_\omega(\chi)}{3}\right), \tag{8}$$

$$\hat{a}(s)=\frac{1}{2}\left(\frac{T_{\min}(\chi)-F_{\min}(\chi)+T_{\max}(\chi)-F_{\max}(\chi)}{2+T_\omega(\chi)-F_\omega(\chi)}\right), \tag{9}$$

$$\hat{c}(s)=\frac{1}{2}\left(\frac{T_{\min}(\chi)+T_{\max}(\chi)}{2+T_\omega(\chi)}\right). \tag{10}$$

where $\hat{s}(s)\in[0,1]$, $\hat{a}(s)\in[-1,1]$, and $\hat{c}(s)\in[0,1]$.

## 3. Operational Principles for NCFNs

Assume two non-empty collections of NCFNs as stated below:

$$\left.\begin{array}{l} A_t(a)=\left\{\left[T_{\min t},T_{\max t}\right],\left[I_{\min t},I_{\max t}\right],\left[F_{\min t},F_{\max t}\right];\left\langle T_{\omega t},I_{\omega t},F_{\omega t}\right\rangle \mid a\in X\right\} \\ A_V(b)=\left\{\left[T_{\min V},T_{\max V}\right],\left[I_{\min V},I_{\max V}\right],\left[F_{\min V},F_{\max V}\right];\left\langle T_{\omega V},I_{\omega V},F_{\omega V}\right\rangle \mid b\in Y\right\} \end{array}\right\}. \tag{11}$$

The following fundamental operational rules such as algebraic sum, algebraic product, union, intersection, and compliment can be defined in [45]:

$$A_t(a)\oplus A_v(b)=\left(\begin{array}{c}\left[T_{\min t}+T_{\min v}-T_{\min t}T_{\min v},T_{\max t}+T_{\max v}-T_{\max t}T_{\max v}\right], \\ \left[I_{\min t}I_{\min v},I_{\max t}I_{\max v}\right],\left[F_{\min t}F_{\min v},F_{\max t}F_{\max v}\right]; \\ \left\langle T_{\omega t}+T_{\omega v}-T_{\omega t}T_{\omega v},I_{\omega t}I_{\omega v},F_{\omega t}F_{\omega v}\right\rangle\end{array}\right), \tag{12}$$

$$A_t(a)\otimes A_v(b)=\begin{pmatrix}\left[T_{\min t}T_{\min v},T_{\max t}T_{\max v}\right],\left[I_{\min t}+I_{\min v}-I_{\min t}I_{\min v},I_{\max t}+I_{\max v}-I_{\max t}I_{\max v}\right],\\ \left[F_{\min t}+F_{\min v}-F_{\min t}F_{\min v},F_{\max t}+F_{\max v}-F_{\max t}F_{\max v}\right];\\ \left\langle T_{\omega t}T_{\omega v},I_{\omega t}+I_{\omega v}-I_{\omega t}I_{\omega v},F_{\omega t}+F_{\omega v}-F_{\omega t}F_{\omega v}\right\rangle\end{pmatrix}, \tag{13}$$

$$A_t(a)\cup A_v(b)=\begin{pmatrix}\left[\max\left(T_{\min t},T_{\min v}\right),\max\left(T_{\max t},T_{\max v}\right)\right],\\ \left[\min\left(I_{\min t},I_{\min v}\right),\min\left(T_{\max t},T_{\max v}\right)\right],\\ \left[\min\left(F_{\min t},F_{\min v}\right),\min\left(F_{\max t},F_{\max v}\right)\right];\\ \left\langle\max\left(T_{\omega t},T_{\omega v}\right),\min\left(T_{\omega t},T_{\omega v}\right),\min\left(T_{\omega t},T_{\omega v}\right)\right\rangle\end{pmatrix}, \tag{14}$$

$$A_t(a)\cap A_v(b)=\begin{pmatrix}\left[\min\left(T_{\min t},T_{\min v}\right),\min\left(T_{\max t},T_{\max v}\right)\right],\\ \left[\max\left(I_{\min t},I_{\min v}\right),\max\left(T_{\max t},T_{\max v}\right)\right],\\ \left[\max\left(F_{\min t},F_{\min v}\right),\max\left(F_{\max t},F_{\max v}\right)\right];\\ \left\langle\min\left(T_{\omega t},T_{\omega v}\right),\max\left(I_{\omega t},I_{\omega v}\right),\max\left(F_{\omega t},F_{\omega v}\right)\right\rangle\end{pmatrix}, \tag{15}$$

$$A_t^{c}(a)=\begin{pmatrix}\left[F_{\min t},F_{\max t}\right],\left[1-I_{\max t},1-I_{\min t}\right],\left[T_{\min t},T_{\max t}\right];\\ \left\langle F_{\omega t},1-I_{\omega t},T_{\omega t}\right\rangle\end{pmatrix}. \tag{16}$$

## 4. Aggregating Operators Established over NCFNs

This section presents a detailed and comprehensive analysis of some aggregating operators, which are based on the NCFNs. It is worth noting that fuzzy-based aggregation operators have a considerable role in MCDM, specifically, in handling subjective preferences, especially when criteria are imprecisely defined. Several fuzzy-based aggregation operators were explored by many researchers like the arithmetic mean, weighted average, geometric mean fuzzy max-min, which offer different ways to handle decision information. It is important to know that each aggregation operator has unique properties that make it suitable for specific types of MCDM problems, providing decision-makers with tailored tools for diverse applications.

### *4.1. Arithmetic Bonferroni Mean Operators*

Bonferroni was the first to propose the concept of the Bonferroni mean. It is a really useful method that may be utilized in a variety of situations. In an MCDM environment, arithmetic Bonferroni mean operators are considered the most powerful tools for solving complex problems, where criteria are interrelated or need a flexible or balanced aggregation approach. It can handle redundancies, provide balanced aggregation, and capture interdependence, which can make it highly effective for complex decision-making scenarios. In this attempt, we developed two novel aggregation operators, namely, neutrosophic cubic fuzzy arithmetic Bonferroni mean and weighted neutrosophic cubic fuzzy Bonferroni arithmetic mean. In addition, by taking full command over these aggregation operators some useful results associated with the neutrosophic cubic fuzzy arithmetic mean along with their meaningful properties are taken into consideration.

**Definition 5.** Let $\alpha_v\left(v=1,\ldots,\kappa\right)$ denote $\gamma_v$ numbers of crisp data. For any $x,y>0,$ the below expression demonstrates the Bonferroni mean operator:

$$B^{x,y}\left(\alpha_1,\ldots,\alpha_\kappa\right)=\left(\frac{1}{\kappa\left(\kappa-1\right)}\sum_{\substack{v,t=1\\v\neq t}}^{\kappa}\alpha_v^x\alpha_t^y\right)^{\frac{1}{x+y}}. \tag{17}$$

By fixing $y=0,$ the operator is reduced to the generalized mean operator as:

$$B^{(x,0)}\left(\alpha_1,\ldots,\alpha_\kappa\right)=\left[\frac{2}{\kappa}\sum_{v=1}^{\kappa}\alpha_v^x\left(\frac{1}{\left(\kappa-1\right)}\sum_{\substack{t=1\\v\neq t}}^{\kappa}\alpha_j^0\right)\right]^{\frac{1}{x+0}}=\frac{1}{\kappa}\left[\sum_{v=1}^{\kappa}\left(\alpha_v^x\right)\right]^{\frac{1}{x}}. \tag{18}$$

Furthermore, by fixing $x=1$ and $y=0,$ the operator took the more comprehensive form, which is presented below:

$$B^{(1,0)}\left(\alpha_1,\ldots,\alpha_\kappa\right)=\frac{1}{\kappa}\left[\sum_{v=1}^{\kappa}\left(\alpha_v^x\right)\right]. \tag{19}$$

**Definition 6.** Suppose $x,y>0$ and $\alpha_i\left(i=1,2,\ldots,\kappa\right)$ be the *n* numbers of crisp numbers. If:

$$AB^{(x,y)}\left(\alpha_1,\ldots,\alpha_\kappa\right)=\frac{1}{x+y}\left[\bigcup_{v,t=1|v\neq t}^{\kappa}\left(x\alpha_v+y\alpha_t\right)^{\frac{1}{\kappa(\kappa-1)}}\right], \tag{20}$$

then the expression stated by $AB^{(x,y)}$ gives the arithmetic Bonferroni mean (AB). The AB has the following characteristics, which are presented in bullets below.

i. $AB^{(x,y)}(0,0,\ldots,0)=0.$

ii. $AB^{(x,y)}\left(\alpha_1,\ldots,\alpha_\kappa\right)=\alpha$ if $\alpha=\alpha_v$ for all $v=1,\ldots,\kappa.$

iii. $AB^{(x,y)}\left(\alpha_1,\ldots,\alpha_\kappa\right)\geq AB^{(x,y)}\left(z_1,\ldots,z_\kappa\right)$ if $\alpha_v\geq z_v$ for all *i*; that is $AB^{(x,y}$ is monotone.

iv. $Min\left(\alpha_v\right)\geq AB^{(x,y)}\geq Max\left(\alpha_v\right).$

### *4.2. Neutrosophic Cubic Fuzzy Arithmetic Bonferroni Mean*

This subsection contains some more valuable qualities of the neutrosophic cubic fuzzy arithmetic Bonferroni operator. Major theoretical results and their proofs are added which will be beneficial for readers for better understanding.

**Definition 7.** Suppose $S$ denotes the collection of NCFNs, then for any real number $x,y>0,$ the expression stated by Eq. (21) represents the neutrosophic cubic fuzzy arithmetic Bonferroni mean operator:

$$NCFBAM^{(x,y)}\left(S_1,\ldots,S_\kappa\right)=\frac{1}{x+y}\left[\underset{v,t=1|v\neq t}{\overset{\kappa}{\oplus}}\left(xS_v\oplus yS_t\right)^{\frac{1}{\kappa(\kappa-1)}}\right]. \tag{21}$$

**Theorem 1:** Consider a non-empty collection of NCFNs as given below:

$$S_v=\left\{\left[T_{v\min},T_{v\max}\right],\left[I_{v\min},I_{v\max}\right],\left[F_{v\min},F_{v\max}\right];\left\langle T_{v\omega},I_{v\omega},F_{v\omega}\right\rangle\right\}, \text{with } x,y>0. \tag{22}$$

Then, the $NCFABM^{(x,y)}$ operator is utilized to evaluate the aggregated value as below:

$$\begin{aligned}
&NCFABM^{(x,y)}\left(S_1,\ldots,S_\kappa\right)=\frac{1}{x+y}\left[\underset{v,t=1|v\neq t}{\overset{\kappa}{\oplus}}\left(xS_v\oplus yS_t\right)^{\frac{1}{\kappa(\kappa-1)}}\right]\\
&=\left\langle\begin{array}{c}
\left[1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-\left(T_{v\min}\right)^x\left(T_{t\min}^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}},1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-\left(T_{v\max}\right)^x\left(T_{t\max}^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}\right],\\
\left[1-\left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-\left(1-I_{v\min}\right)^x\left(1-I_{t\min}\right)^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}},\ 1-\left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-\left(1-I_{v\max}\right)^x\left(1-I_{t\max}\right)^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}\right],\\
\left[1-\left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-\left(1-F_{v\min}\right)^x\left(1-F_{t\min}\right)^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}},1-\left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-\left(1-F_{v\max}\right)^x\left(1-F_{t\max}\right)^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}\right],\\
\left\langle\begin{array}{c}1-\left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-\left(T_{v\omega}\right)^x\left(T_{t\omega}\right)^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}},\left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-\left(1-I_{v\omega}\right)^x\left(1-I_{t\omega}\right)^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}\\
\left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-\left(1-F_{v\omega}\right)^x\left(1-F_{t\omega}\right)^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}\end{array}\right\rangle
\end{array}\right\rangle
\end{aligned} \tag{23}$$

**Proof of Theorem 1:** The NCFNs' operational laws allow us to write:

$$xS_v=\left\{\begin{array}{c}\left[1-\left(1-T_{\min v}\right)^x,1-\left(1-T_{\max v}\right)^x\right],\left[\left(I_{\min v}\right)^x,\left(I_{\max v}\right)^x\right],\left[\left(F_{\min v}\right)^x,\left(F_{\max v}\right)^x\right];\\
\left\langle 1-\left(1-T_{\omega v}\right)^x,\left(I_{\omega v}\right)^x,\left(F_{\omega v}\right)^x\right\rangle\end{array}\right\},$$

$$yS_t = \left\{ \begin{array}{c} \left[1-\left(1-T_{\min t}\right)^y, 1-\left(1-T_{\max t}\right)^y\right], \left[\left(I_{\min t}\right)^y, \left(I_{\max t}\right)^y\right], \left[\left(F_{\min t}\right)^y, \left(F_{\max t}\right)^y\right]; \\ \left\langle 1-\left(1-T_{\omega t}\right)^y, \left(I_{\omega t}\right)^y, \left(F_{\omega t}\right)^y \right\rangle \end{array} \right\}.$$

So:

$$xS_v \oplus yS_t = \left\{ \begin{array}{l} \left[1-\left(1-T_{\min v}\right)^x \left(1-T_{\min t}\right)^y, 1-\left(1-T_{\max v}\right)^x \left(1-T_{\max t}\right)^y\right], \\ \left[\left(I_{\min v}\right)^x \left(I_{\min t}\right)^y, \left(I_{\max v}\right)^x \left(I_{\max t}\right)^y\right], \left[\left(F_{\min v}\right)^x \left(F_{\min t}\right)^y, \left(F_{\max v}\right)^x \left(F_{\max t}\right)^y\right]; \\ \left\langle 1-\left(1-T_{\omega v}\right)^x \left(1-T_{\omega t}\right)^y, \left(I_{\omega v}\right)^x \left(I_{\omega t}\right)^y, \left(F_{\omega v}\right)^x \left(F_{\omega t}\right)^y \right\rangle \end{array} \right\}.$$

In consequence, we have:

$$\underset{v,t=1|v\neq t}{\overset{\kappa}{\oplus}} \left(xS_v \oplus yS_t\right)^{\frac{1}{\kappa(\kappa-1)}}$$

$$= \left\langle \begin{array}{c} \left[1-\bigcup_{\substack{v,t=1\\v\neq t}}^{\kappa}\left(1-(T_{v\min})^x\left(T^y_{t\min}\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}, 1-\bigcup_{\substack{v,t=1\\v\neq t}}^{\kappa}\left(1-(T_{v\max})^x\left(T^y_{t\max}\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}\right], \\ \left[1-\left(1-\bigcup_{\substack{v,t=1\\v\neq t}}^{\kappa}\left(1-(1-I_{v\min})^x(1-I_{t\min})^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}, 1-\left(1-\bigcup_{\substack{v,t=1\\v\neq t}}^{\kappa}\left(1-(1-I_{v\max})^x(1-I_{t\max})^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}\right], \\ \left[1-\left(1-\bigcup_{\substack{v,t=1\\v\neq t}}^{\kappa}\left(1-(1-F_{v\min})^x(1-F_{t\min})^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}, 1-\left(1-\bigcup_{\substack{v,t=1\\v\neq t}}^{\kappa}\left(1-(1-F_{v\max})^x(1-F_{t\max})^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}\right], \\ \left\langle \begin{array}{c} 1-\left(1-\bigcup_{\substack{v,t=1\\v\neq t}}^{\kappa}\left(1-(T_{v\omega})^x(T_{t\omega})^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}, \left(1-\bigcup_{\substack{v,t=1\\v\neq t}}^{\kappa}\left(1-(1-I_{v\omega})^x(1-I_{t\omega})^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}, \\ \left(1-\bigcup_{\substack{v,t=1\\v\neq t}}^{\kappa}\left(1-(1-F_{v\omega})^x(1-F_{t\omega})^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}} \end{array} \right\rangle \end{array} \right\rangle.$$

This equation now yields the NCFABM operator $NCFABM^{x,y}\left(S_1, S_2, \ldots, S_\kappa\right)$ utilizing the operational principles of neutrosophic cubic sets imparted as per the above equation. Besides, it also meets the following requirements:

$$\left[1-\bigcup_{v,t=1|v\neq t}^{\kappa}\left(1-(T_{v\min})^x\left(T^y_{t\min}\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}, 1-\bigcup_{v,t=1|v\neq t}^{\kappa}\left(1-(T_{v\max})^x\left(T^y_{t\max}\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}\right] \subseteq [0,1],$$

$$\left[1-\left(1-\underset{v,t=1|v\neq t}{\overset{\kappa}{\cup}}\left(1-(1-I_{v\min})^{x}(1-I_{t\min})^{y}\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}},1-\left(1-\underset{v,t=1|v\neq t}{\overset{\kappa}{\cup}}\left(1-(1-I_{v\max})^{x}(1-I_{t\max})^{y}\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}\right]\subseteq[0,1],$$

$$\left[1-\left(1-\underset{v,t=1|v\neq t}{\overset{\kappa}{\cup}}\left(1-(1-F_{v\min})^{x}(1-F_{t\min})^{y}\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}},1-\left(1-\underset{v,t=1|v\neq t}{\overset{\kappa}{\cup}}\left(1-(1-F_{v\max})^{x}(1-F_{t\max})^{y}\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}\right]\subseteq[0,1],$$

$$0\leq 1-\left(1-\underset{v,t=1|v\neq t}{\overset{\kappa}{\cup}}\left(1-(T_{v\omega})^{x}(T_{t\omega})^{y}\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}\leq 1,$$

$$0\leq\left(1-\underset{v,t=1|v\neq t}{\overset{\kappa}{\cup}}\left(1-(1-I_{v\omega})^{x}(1-I_{t\omega})^{y}\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}\leq 1.$$

Finally, we get the below relation:

$$0\leq\left(1-\underset{\substack{v,t=1\\ v\neq t}}{\overset{\kappa}{\cup}}\left(1-(1-F_{v\omega})^{x}(1-F_{t\omega})^{y}\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}\leq 1.$$

### *4.3. Efficient and Desirable Features*

Some important and useful theorems and their proofs are presented in this part. These theorems include idempotency, commutativity, and boundedness.

**Theorem 2 (idempotency):** Consider $S=\{[T_{\min},T_{\max}],[I_{\min},I_{\max}],[F_{\min},F_{\max}];\langle T_{\omega},I_{\omega},F_{\omega}\rangle\}$ be the non-empty collection of NCF numbers and $x,y>0$. Then, by aggregation operator, we are obliged to record:

$$\begin{aligned}&NCFABM^{x,y}(S_1,\dots,S_\kappa)=NCFABM^{x,y}(S,\dots,S)=\frac{1}{x+y}\left(\underset{v,t=1|v\neq t}{\overset{\kappa}{\oplus}}(xS\oplus yS)^{\frac{1}{\kappa(\kappa-1)}}\right)\\&=\frac{1}{x+y}\left(\underset{v,t=1|v\neq t}{\overset{\kappa}{\oplus}}\left(S\left(x\oplus y\right)\right)^{\frac{1}{\kappa(\kappa-1)}}\right)=\frac{1}{x+y}\left((S(x\oplus y))\frac{\kappa(\kappa-1)}{\kappa(\kappa-1)}\right)=S.\end{aligned}\qquad(23)$$

**Theorem 3 (commutativity):** Consider *S* be the non-empty collection of NCF numbers and $x,y>0$. Then, by aggregation operator, we are obligated to draft that $NCFABM^{x,y}(S_1,\dots,S_\kappa)=NCFABM^{x,y}\left(S_1^{'},\dots,S_\kappa^{'}\right)$, where $\left(S_1^{'},\dots,S_\kappa^{'}\right)$ is any permutation of

$(S_1,...,S_\kappa)$. Then:

$$NCFABM^{x,y}(S_1,...,S_\kappa)=\frac{1}{x+y}\left(\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\oplus}}(xS\oplus yS)\frac{1}{\kappa(\kappa-1)}\right)=\frac{1}{x+y}\left(\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\oplus}}(xS'\oplus yS')\frac{1}{\kappa(\kappa-1)}\right)=NCFABM^{x,y}(S'_1,...,S'_\kappa). \quad (24)$$

**Theorem 4 (boundedness):** Consider $S=\{[T_{\min},T_{\max}],[I_{\min},I_{\max}],[F_{\min},F_{\max}];\langle T_\omega,I_\omega,F_\omega\rangle\}$ be the non-empty collection of NCF numbers and $x,y>0$. Suppose that

$$S_V^-=\begin{Bmatrix}\inf([T_{V\min},T_{V\max}]),\sup([I_{V\min},I_{V\max}]),\sup([F_{V\min},F_{V\max}]);\\ \langle\min T_{V\omega},\max I_{V\omega},\max F_{V\omega}\rangle\end{Bmatrix}, \quad (25)$$

$$S_V^+=\begin{Bmatrix}\sup([T_{V\min},T_{V\max}]),\inf([I_{V\min},I_{V\max}]),\inf([F_{V\min},F_{V\max}]);\\ \langle\max T_{V\omega},\min I_{V\omega},\min F_{V\omega}\rangle\end{Bmatrix}. \quad (26)$$

For any $x,y>0$ then $S_V^-\leq NCFABM^{x,y}(S_1,...,S_\kappa)\leq S_V^+$ gives the boundedness. If parameters *x* and *y* are changed, the specific case can be achieved in $NCFABM^{x,y}$. If $y\to 0$, then we may write:

$$NCFABM^{x,y}(S_1,...,S_\kappa)=\frac{1}{x+y}\left(\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\oplus}}(xS\oplus yS)^{\frac{1}{\kappa(\kappa-1)}}\right)=\frac{1}{x}\left(\underset{v=1}{\overset{\kappa}{\oplus}}(xS_V)\frac{1}{\kappa}\right)$$

$$=\left\langle\begin{matrix}\left[1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-(T_{V\min})^{\frac{1}{x}}\right)^{\frac{1}{x}},1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-(T_{t\max})^{\frac{1}{x}}\right)^{\frac{1}{x}}\right],\\ \left[1-\left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-(1-I_{V\min})^{x}\right)^{\frac{1}{x}}\right)^{\frac{1}{x}},1-\left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-(1-I_{V\min})^{x}\right)^{\frac{1}{x}}\right)^{\frac{1}{x}}\right],\\ \left[1-\left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-(1-F_{V\min})^{x}\right)^{\frac{1}{x}}\right)^{\frac{1}{x}},1-\left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-(1-F_{V\min})^{x}\right)^{\frac{1}{x}}\right)^{\frac{1}{x}}\right],\\ \left\langle1-\left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-(T_{V\omega})^{x}\right)^{\frac{1}{x}}\right)^{\frac{1}{x}},\left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-(1-I_{V\omega})^{x}\right)^{\frac{1}{x}}\right)^{\frac{1}{x}},\left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-(1-F_{V\omega})^{x}\right)^{\frac{1}{x}}\right)^{\frac{1}{x}}\right\rangle\end{matrix}\right\rangle. \quad (27)$$

The term generalized neutrosophic cubic fuzzy arithmetic mean $NCFABM^{x,y}$ will be used.

## 5. Weighted Neutrosophic Cubic Fuzzy Bonferroni Arithmetic Mean

Weighted aggregation operators remain to be crucial in several mathematical procedures, especially in MCDM processes. We put forward a weighted aggregation operator established upon the neutrosophic cubic fuzzy Bonferroni arithmetic mean, which is delineated as follows.

**Definition 8.** Consider a non-empty collection of NCF numbers. The associated weight vector with $S_v = (s_1,...,s_\kappa)$ are $\varpi = (\varpi_1,...,\varpi_\kappa)^T$, whereas $\varpi_v$ represent the prominence level of $s_v$ such that each $\varpi_v > 0$ and $\sum_{v=1}^{\kappa}\varpi_v = 1.$ Then, for any $x, y > 0,$ weighted neutrosophic cubic fuzzy arithmetic Bonferroni mean operator is:

$$NCFGAM^{x,y}(s_1,...,s_\kappa) = \frac{1}{x+y}\left(\underset{v,t=1|v\neq t}{\overset{\kappa}{\oplus}}\left(xs_v \oplus ys_t\right)^{\frac{1}{\kappa(\kappa-1)}}\right). \tag{28}$$

**Theorem 5.** Consider a non-empty collection of NCF numbers. The associated weight vector with $S_v = (s_1,...,s_\kappa)$ are $\varpi = (\varpi_1,...,\varpi_\kappa)^T$, whereas $\varpi_v$ represent the prominence level of $s_v$ such that each $\varpi_v > 0$ and $\sum_{v=1}^{\kappa}\varpi_v = 1.$ The aggregated value is then computed as:

$$NCFGAM_{\omega}^{x,y} = \frac{1}{x+y}\left(\underset{v,t=1|v\neq t}{\overset{\kappa}{\bigoplus}}\left(x\left(\breve{S}_v\right)^{\omega_v} \oplus y\left(\breve{S}_t\right)^{\omega_t}\right)^{\frac{1}{\kappa(\kappa-1)}}\right)$$

$$= \left\langle \begin{array}{c} \left[1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-\left((T_{v\min})^{\varpi_v}\right)^x\left(\left(T_{t\min}^{y}\right)^{\varpi_t}\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}, 1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-\left((T_{v\max})^{\varpi_v}\right)^x\left(\left(T_{t\max}^{y}\right)^{\varpi_t}\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}\right], \\ \left[1-\left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-\left((1-I_{v\min})^{\varpi_v}\right)^x\left((1-I_{t\min})^{\varpi_t}\right)^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}, 1-\left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-\left((1-I_{v\max})^{\varpi_v}\right)^x\left((1-I_{t\max})^{\varpi_t}\right)^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}\right], \\ \left[1-\left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-\left((1-F_{v\min})^{\varpi_v}\right)^x\left((1-F_{t\min})^{\varpi_t}\right)^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}, 1-\left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-\left((1-F_{v\max})^{\varpi_v}\right)^x\left((1-F_{t\max})^{\varpi_t}\right)^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}\right], \\ \left\langle \begin{array}{c} 1-\left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-\left((T_{v\omega})^{\varpi_t}\right)^x\left((T_{t\omega})^{\varpi_t}\right)^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}, \left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-\left((1-I_{v\omega})^{\varpi_v}\right)^x\left((1-I_{t\omega})^{\varpi_t}\right)^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}}, \\ \left(1-\underset{\substack{v,t=1\\v\neq t}}{\overset{\kappa}{\cup}}\left(1-\left((1-F_{v\omega})^{\varpi_v}\right)^x\left((1-F_{t\omega})^{\varpi_t}\right)^y\right)^{\frac{1}{\kappa(\kappa-1)}}\right)^{\frac{1}{x+y}} \end{array} \right\rangle \end{array} \right\rangle. \tag{29}$$

Proof is omitted because following the above theorems it can be done easily. Some other related theorems can also easily be proved analogous to the proofs rendered in the previous section.

## 6. An Approach to MCDM using Proposed Operator in Different Scenarios

This part is devoted to the use of suggested operators in the MCDM problem. We presented a numerical example, which is based on neutrosophic cubic fuzzy data. A short overview of the procedure is made in the algorithm as given below.

### *6.1. Algorithm*

Consider a finite collection of *P* choices; i.e. $A=\{\delta_1,...,\delta_p\}$ with a finite collection of *q* attributes; i.e. $C=\{\rho_1,...,\rho_q\}$. The associated weight vector is $(\bar{\omega})^T=\{\bar{\omega}_1,\bar{\omega}_2,\bar{\omega}_3,...,\bar{\omega}_q\}$ such that $\bar{\omega}>0$ and $\Sigma\bar{\omega}=1$. The essential steps are given below.

**Step 1.** Initially a NCF decision matrix $D=\left[N_{ij}\right]_{p\times q}$ is formed.

**Step 2.** The obtained matrix is securitized via $NCFABM_{\bar{\omega}}^{\varphi,\phi}$ to *q* characteristics.

**Step 3.** Score formula is utilized to compute the score values by using score formula. The score formula approach is used to compute the values for each alternative.

**Step 4.** The *P* choices are ranked based on computed scores.

### *6.2. Numerical Example and Investigation*

Ayub Medical and Teaching Hospital, situated in the beautiful city of Abbottabad, Pakistan, is not just the biggest single-site emergency clinic on the planet yet in addition a main restorative focus that treats different confounded and extreme cases, particularly in the fields of life sciences mainly comprising living giver liver transplantation, clinical anesthesia, and serious intense pancreatitis. The hospital is an extensive emergency clinic categorized as Grade A Class Three in Pakistan.

The hospital is likewise trying to turn into a countrywide place for innovation development and medicinal logical research. It facilitates a massive number of patients from a capital territory, encompassing urban territories and rural communities because of its propelled medicinal innovation, gear, and abundance of restorative assets. As indicated by the medical clinic's very own measurements, there are around 18,000 non-admitted patients and exceeding 6,000 patients pausing for a bed on some random day. As of late, proceeding with the improvement of the hospital has prompted the development of some administration issues. We realize that beds are an amazingly rare emergency clinic asset and that the primary objective of medical clinic bed executives is to boost the usage of assets. Consistently, the WCH concedes almost 200,000 inpatients, yet in any event, 30% of this number (around 60,000 inpatients) should not be treated in the hospital. A small extent of the inpatients experience extreme ailment or exceptional estimation of clinical pathology. However, the greater part of them experience the ill effects of less genuine conditions; e.g. minor injury and toothache. These patients are embracing an excellent therapeutic asset, which implies that there are no beds for other patients requiring clinical treatment. Then again, treating patients with straightforward infections (a ruptured appendix and gallbladder issues) permits the hospital to take an interest being developed of restorative innovation and logical research.

The contention amongst the huge number of patients chasing the fantastic assets of the hospital despite having no clinical requirement for them and the shortage of restorative assets has become an increasingly major issue for the hospital. The hospital confirmation focus was set up in 2011 to manage this precarious issue. The inside has two fundamental goals:

i. to advance the hospitalization information process and select patients,
ii. to enhance the productivity with which assets are utilized and guarantee the reasonable and wise utilization of restorative assets.

The primary phase of the inpatient information process is that the patient is analyzed in an outpatient and at that point enrolled by the confirmation focus to save holding up outside the medical clinic. The framework is intended to organize information on patients who have dire need of a bed or the most genuine clinical treatment. The prioritization of patients should resolve the contention between the inventory of medical clinic assets and the extensive interest of patients. Consistently, there are numerous patients and information focus. Positioning the patients proves to be a difficult task, given exceptionally emotional criteria. We in this way need an effective, experimentally based choice help strategy to help the works and choose which patients ought to be organized, which patients should hang tight for information, and which patients ought not to be conceded.

The criteria are:

i. *Estimation of clinical pathology* ($C_1$) – Managing patients who experience the ill effects of troublesome or entangled sickness can advance the improvement of astrology and have a high estimation of clinical pathology.
ii. *Clinical features* ($C_2$) – Speaks to sickness seriousness, agony, and impediments on action.
iii. *Urgency* ($C_3$) – Patients who are genuinely sick or have a condition requiring crisis treatment are delegated having high criticalness.

For straightforwardness of introduction, we will utilize three patients to show the proposed strategy. We welcomed medical clinic specialists (for example, the outpatient specialist and staff in the confirmation focus) to analyze these patients and give their appraisals in IMSs. For example, the master may think that the patient worth (because of information criteria) is between extraordinary acknowledgment and solid acknowledgment.

The decision experts offer the assessment values $(\varpi)^T = (0.4, 0.22, 0.38)$ of preferable alternatives $A_i\ (i=1,2,3)$ having $C_j\ (j=1,2,3)$ in the form of NCFNs, which are proposed weight vector for the specified characteristics. Consequently, a neutrosophic cubic decision matrix $\overline{D} = \left[\overline{S}_{ij}\right]_{p\times q}$ may be used to represent the necessary evaluation data.

**Step 1.** Neutrosophic cubic fuzzy decision matrix $\overline{D} = \left[\overline{S}_{ij}\right]_{3\times 3}$ is formed as displayed in Table 1.

**Step 2:** To substitute different values of the pair $(\varphi, \phi)$ makes distinct cases therefore the obtained cubic fuzzy decision matrix is aggregated against each case by using neutrosophic cubic fuzzy arithmetic Bonferroni means aggregation and then the resulting outcomes are passed through score function as shown in Table 2.

**Step 3.** The alternative rankings are made by focusing on the chosen attributes, which are showcased in Table 3 below.

**Table 1**
Decision matrix based on neutrosophic cubic fuzzy data

| | ***C*₁** | ***C*₂** | ***C*₃** |
|---|---|---|---|
| ***A*₁** | $\begin{pmatrix}[0.32,0.42],\\ [0.57,0.32],\\ [0.14,0.34];\\ \langle 0.43,0.36,0.67\rangle\end{pmatrix}$ | $\begin{pmatrix}[0.45,0.32],\\ [0.75,0.47],\\ [0.36,0.78];\\ \langle 0.87,0.36,0.98\rangle\end{pmatrix}$ | $\begin{pmatrix}[0.89,0.92],\\ [0.36,0.43],\\ [0.43,0.45];\\ \langle 0.89,0.65,0.32\rangle\end{pmatrix}$ |
| ***A*₂** | $\begin{pmatrix}[0.46,0.45],\\ [0.97,0.87],\\ [0.54,0.62];\\ \langle 0.17,0.35,0.68\rangle\end{pmatrix}$ | $\begin{pmatrix}[0.47,0.89],\\ [0.43,0.23],\\ [0.45,0.32];\\ \langle 0.23,0.75,0.34\rangle\end{pmatrix}$ | $\begin{pmatrix}[0.23,0.78],\\ [0.35,0.47],\\ [0.57,0.67];\\ \langle 0.27,0.57,0.42\rangle\end{pmatrix}$ |
| ***A*₃** | $\begin{pmatrix}[0.34,0.54],\\ [0.67,0.87],\\ [0.56,0.67];\\ \langle 0.45,0.47,0.62\rangle\end{pmatrix}$ | $\begin{pmatrix}[0.67,0.78],\\ [0.89,0.67],\\ [0.23,0.98];\\ \langle 0.45,0.89,0.23\rangle\end{pmatrix}$ | $\begin{pmatrix}[0.87,0.67],\\ [0.89,0.45],\\ [0.78,0.89];\\ \langle 0.34,0.56,0.78\rangle\end{pmatrix}$ |

**Table 2**
Score function for distinct cases of parametric values

| ***Parameter values*** | $\overline{A}_i = NCFABM\ (i = 1,\ldots,4)$ | ***Score values*** |
|---|---|---|
| | *Case 1* | |
| ***φ*=*ϕ*=0.5** | $\{[.014,0.057],[.007,.021],[.002,.043];\langle .033,.008,.078\rangle\}$ | **0.5872** |
| | $\{[.002,.047],[.063,.007],[.003,.091];\langle .038,.007,.005\rangle\}$ | **0.6540** |
| | $\{[.003,.045],[.023,.069],[.021,.057];\langle .389,0.078,.305\rangle\}$ | **0.3872** |
| | ***Case 2*** | |
| ***φ*=*ϕ*=1** | $\{[.017,.040],[.013,.020],[.013,0.028];\langle .089,.037,.435\rangle\}$ | **0.3762** |
| | $\{[.013,.094],[.043,.067],[.023,.028];\langle .037,.023,.012\rangle\}$ | **0.4279** |
| | $\{[.045,.081],[0.053,0.079],[0.019,.073];\langle .084,.031,.042\rangle\}$ | **0.3041** |
| | ***Case 3*** | |
| ***φ*=1, *ϕ*=0.5** | $\{[.017,.040],[.013,.020],[.013,0.028];\langle .089,.037,.435\rangle\}$ | **0.5390** |
| | $\{[.017,.040],[.013,.020],[.013,0.028];\langle .089,.037,.435\rangle\}$ | **0.7520** |
| | $\{[.017,.040],[.013,.020],[.013,0.028];\langle .089,.037,.435\rangle\}$ | **0.4523** |
| | ***Case 4*** | |
| ***φ*=0.5, *ϕ*=1** | $\{[.013,.019],[.046,.051],[.021,.038];\langle .039,.023,.195\rangle\}$ | **0.4190** |
| | $\{[.021,.034],[.041,.062],[.024,.035];\langle .021,.032,.057\rangle\}$ | **0.4672** |
| | $\{[.013,.020],[.032,.047],[.051,.059];\langle .052,.012,.023\rangle\}$ | **0.4042** |

**Table 3**
Hierarchy of choices

| **Cases** | **Parameter values** | **Rankings** |
|---|---|---|
| **Case 1** | ***φ*=*ϕ*=0.5** | $\overline{A}_2 \succ \overline{A}_1 \succ \overline{A}_3$ |
| **Case 2** | ***φ*=*ϕ*=1** | $\overline{A}_2 \succ \overline{A}_1 \succ \overline{A}_3$ |
| **Case 3** | ***φ*=1, *ϕ*=0.5** | $\overline{A}_2 \succ \overline{A}_1 \succ \overline{A}_3$ |
| **Case 4** | ***φ*=0.5, *ϕ*=1** | $\overline{A}_2 \succ \overline{A}_1 \succ \overline{A}_3$ |

### *6.3. Results and Discussion*

The current investigation delved into some novel neutrosophic cubic fuzzy-based aggregation operators, which are crucial in many real-life situations. The aggregating operators based on NCFSs play a pivotal role in MCDM by aggregating evaluations or preferences by accommodating incomplete opinions, involving different criteria, and ambiguous judgments. For example, by using NCFS aggregation, the decision-maker can evaluate suppliers through indeterminate judgments or uncertain data in supply chain management, criteria like delivery, cost, quality, and time might have subjective evaluations. Similarly, several other treatments could be formed by using the neutrosophic cubic fuzzy-based aggregation operators. This key attention fetched this study, which deeply looked into cracking neutrosophic cubic fuzzy data by two novel aggregation operators; i.e. arithmetic Bonferroni mean and weighted arithmetic Bonferroni mean operator. In addition, we focalized the aggregation of neutrosophic cubic fuzzy information by focusing on different parametric values of parameters $(\varphi,\phi)$, which insights into the assessment of aggregation of large neutrosophic cubic fuzzy data in a more comprehensive manner and are very effective in the evaluation of many MCDM problems. The solidity and reliability of the proposed operators are analyzed through a numerical example. In the current example, the alternatives are evaluated against the chosen attributes in different scenarios. Four different cases are generated by varying values of the parameters $(\varphi,\phi)$. The cases are made by assuming the values of the parameters from 0 to 1. The first two cases are observed by the unique value of both parameters, whereas the other two cases are computed for distinct values of parametric values $(\varphi,\phi)$. The score values are computed by employing the score function formula of neutrosophic cubic interval-valued sets, and then ranking is obtained accordingly.

The computed outcome for each case is presented in Figure 1.

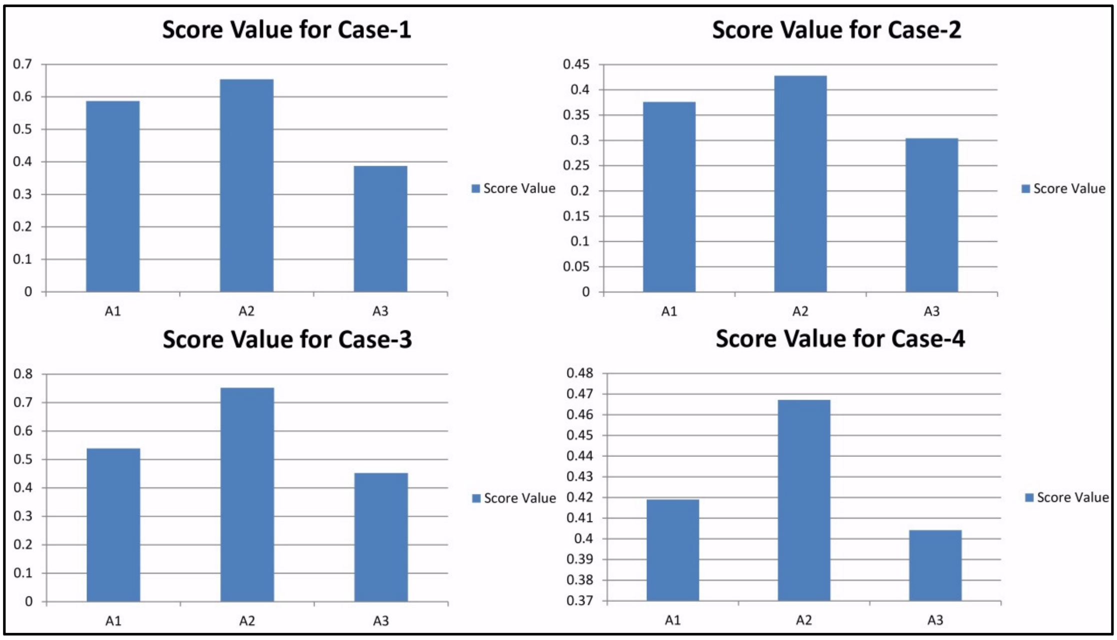
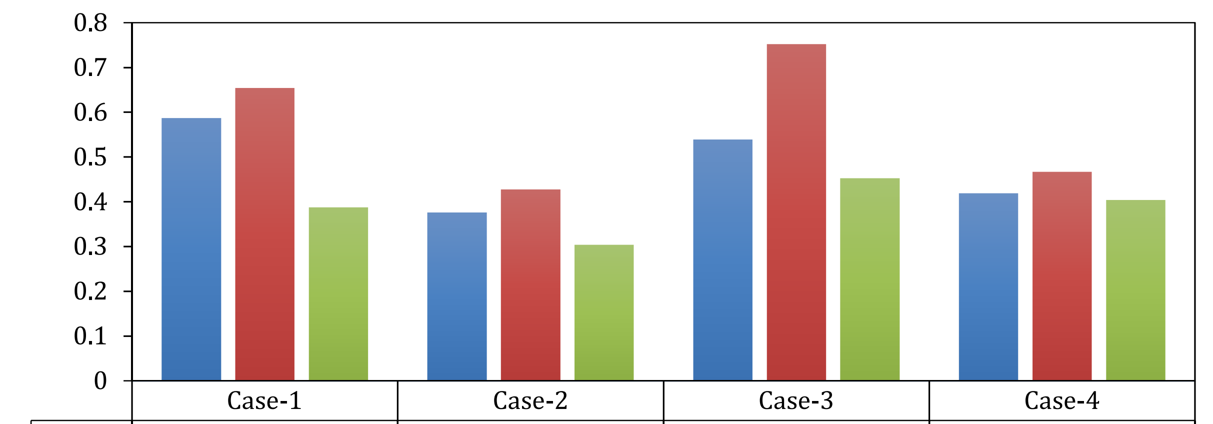



**Fig. 1**. Score values for the alternatives

Figure 2 illustrates the collective analysis of the different cases.




| | Case-1 | Case-2 | Case-3 | Case-4 |
|---|---|---|---|---|
| S(A1) | 0.5872 | 0.3762 | 0.539 | 0.419 |
| S(A2) | 0.654 | 0.4279 | 0.752 | 0.4672 |
| S(A3) | 0.3872 | 0.3041 | 0.4523 | 0.4042 |

**Fig. 2**. Distinct cases of score values

It is observed that the range of the most preferable options did not change and stayed the same in every case despite subjective preferences for various values of the parameters. The computed output for several distinct cases generates more optimistic results, which realize the credibility of the proposed model compared to other existing MCDM aggregation operators. In addition, this conduct could lead us to acknowledge the solidity and reliability of the proposed operator. It is noteworthy that the preservation of alternative rankings with varying values of parameters remains ineffective for the subjective preference of the decision-maker.

The obtained results have a strong association with the study published in [45], which ensures the solidity and viability of the effectiveness and robustness of the suggested aggregating operator. Furthermore, the proposed assessment of alternative rankings is slightly different in the case of varying values of parameters and demonstrates the decision-maker's risk preferences.

On the other hand, the earlier study [48] reflects quite different assessments of alternatives based on weighted Atanassov's intuitionistic fuzzy Bonferroni mean (WIFBM) aggregation operator.

Table 2 illustrates alternative scores for different cases obtained by our method, which are almost greater than zero and near to human cognition while their study fetched the assessment of alternatives smaller than zero. It is pertinent to note that the proposed operators based on neutrosophic cubic fuzzy information produce more optimistic expectations compared to aggregations based on the WIFBM aggregation operator, which gives pessimistic expectations. Thus, the proposed evaluation strictly complies with the optimism and pessimism criteria defined in earlier published studies [49].

Therefore, it should be concluded that the proposed operator can be considered as optimistic while the operators studied in [48] can be considered pessimistic. Likewise, parametric values can be considered as optimistic or pessimistic respectively. NCFABM is not merely an extension of the Bonferroni mean, and the NCFWABM is not solely a generalization of the ABM because of neutrosophic cubic information. It provides summative information which includes the three parts of information; i.e., truth, indeterminacy, and falsity. Moreover, the weights of the criteria for neutrosophic cubic fuzzy-based BM are quite essential in many practical problems, specifically in MCDM. Overall, NCFWAO operators can have more pessimistic findings compared to those of Bonferroni mean and WBM. Therefore, the intensive need for weighted aggregation for neutrosophic cubic fuzzy information initiated the idea of NCFBM and NCFWBM operators, which provide more choices for the decision makers based on varying choices of the parameters.

## 7. Conclusion

Neutrosophic cubic fuzzy information combining the features of single and multiple interval valued neutrosophic sets and cubic fuzzy sets, offer a powerful tool for capturing higher levels of fuzziness, uncertainty, and indeterminacy. Aggregating operators based on single- and multi-valued neutrosophic cubic interval-valued fuzzy sets can tackle the complexity, which usually lies due to uncertainty and fuzziness.

We presented the basic notions of neutrosophic cubic fuzzy sets and then by taking full advantage of the Bonferroni mean, the study focalized on two neutrosophic cubic fuzzy-based aggregation operators. The study is not only limited to aggregating the neutrosophic cubic fuzzy information but is deeply looked into evaluation by the choice of different parametric values. The associated fundamental properties of the new operators are taken into consideration and key features are uncovered for multiple values of parameters. Some necessary theoretical results of the proposed operator are also presented in detail. Furthermore, we build an aggregation method for multiple criteria problems involving neutrosophic cubic numbers. To end with a numerical example is rendered to make evident the pertinence and fitness of the suggested technique in scenarios involving group decision-making.

Diverging perspectives are common in group decision-making difficulties because the specialists typically originate from various specialist areas and experience diverse backgrounds and degrees of expertise. The visual representation of tabulated data also provides an open perspective for challengers and the rest of the job. In the future, we believe this tool might be utilized for a variety of MCDM objectives, which could easily address the complexity of modern decision-making problems.

**Acknowledgment**
This research was not funded by any grant.

**Conflicts of Interest**
Authors declare no conflict of interest in the publication of this research.

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