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Addressing Decision-Making Challenges: Similarity Measures for Interval-Valued Intuitionistic Fuzzy Hypersoft Sets

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ABSTRACT

A similarity measure tackles multiple difficulties, including managing ambiguous and indistinct information. Nevertheless, it encounters difficulties with general ambiguities and complexities in matters requiring heterogeneous data. This article examines essential ideas that establish a framework for the study, including soft sets, hypersoft sets, intuitionistic fuzzy hypersoft sets (IFHSS), and interval-valued IFHSS (IVIFHSS). The main aim of this research is to construct similarity metrics for IVIFHSS and examine their fundamental features. A decision-making methodology is suggested utilizing these similarity measures in conjunction with a strategy for addressing multi-tiered decision-making challenges. The merits of the algorithm, such as its efficiency, adaptability, and relative superiority to traditional methods, are outlined.

1. Introduction

Decision-making is a stimulating challenge centered on identifying the optimal alternative. Although obtaining exact numerical data on alternatives may appear simple, in practical scenarios, collective and comprehensive information frequently demonstrates greater reliability than deceptive or partial data.

Fuzzy sets (FSs) are analogous to classical sets but incorporate a membership degree (MD) for each element. In conventional set theory, items are assessed in a binary fashion to ascertain their membership in a set. The FS theory facilitates sophisticated classification by assigning degrees of membership using a membership function that operates inside the interval [0, 1]. In certain circumstances, decision-makers evaluate MDs and non-membership degrees (NMDs). Nonetheless, Zadeh's FS paradigm [1] is inadequately prepared to address ambiguous or imprecise information. Atanassov proposed the idea of intuitionistic fuzzy sets (IFS) to mitigate these restrictions. IFS

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enhances FS theory by integrating MD and NMD thus offering a more comprehensive framework for handling uncertain and ambiguous information.

Atanassov's IFSs [2] do not adequately handle situations where decision-makers face MD = 0.6 and NMD = 0.7, where MD + NMD $\nleq$ 1. To address this issue, Yager [3, 4] proposed Pythagorean fuzzy sets (PFS), altering the requirement to $MD^2 + NMD^2 \leq 1$. Zhang and Xu [5] expanded on this approach by formulating operational rules for PFS and introducing a decision-making method to address multi-criteria decision-making (MCDM) challenges. Wei and Lu [6] proposed power aggregation operators (AOs) and developed a decision-making technique for MCDM within a Pythagorean fuzzy context. Zhang et al. [7-10] introduced a fuzzy robust control technique for nonlinear supply chain systems characterized by lead times and uncertain demands, employing Takagi-Sugeno fuzzy systems to guarantee robust stability, limit fluctuations, and decrease operational costs via soft subsystem transitions. Furthermore, emergency models and a comprehensive emergency strategy are formulated for disrupted supply chains, enhanced by a sliding mode control method for fuzzy stochastic systems under cyberattacks, demonstrating efficacy through simulations [11-12].

Sarwar and Li [13] found common solutions for second-order nonlinear boundary value problems with fuzzy sets. Peng and Yuan [13] advanced the field by introducing novel operators, including the Pythagorean fuzzy point operator, and developed decision-making procedures utilizing their methodologies. They concentrated on fundamental operations inside PFS, developing critical tools and methodologies for decision-making. Garg [15] formulated logarithmic arithmetic principles for PFS and offered novel AOs, whereas Arora and Garg [16] enhanced linguistic IFS by proposing operational rules and AOs. Ma and Xu [17] introduced novel AOs and developed score and accuracy functions for Pythagorean fuzzy numbers, thereby enhancing the utilization of PFS in decision-making.

Molodtsov [18] established the notion of Soft Sets (SS) to tackle the problems of managing vague, erroneous, and ambiguous items through parameterization. Maji et al. [19] enhanced the SS framework by including essential operators and their characteristics. Maji et al. [20] advanced a decision-making technique employing these operators. Maji et al. [21] integrated the concepts of FS and SS to develop fuzzy soft sets (FSS). Then, Maji et al. [22] presented Intuitionistic fuzzy soft sets (IFSS) by elucidating their essential functionalities. Garg and Arora [23] expanded the IFSS framework by introducing a generalized version incorporating AOs and a decision-making approach to handle uncertain and imprecise IFSS data. Xia et al. [24] examined sampled-data stabilization for chaotic nonlinear systems employing a Takagi-Sugeno fuzzy model. Zulqarnain et al. [25] introduced aggregation operators and correlation coefficients for interval-valued IFSSs and developed a TOPSIS-based methodology to address MCDM challenges. Peng et al. [26] introduced the Pythagorean fuzzy soft sets (PFSS), integrating PFS with SS, and defined their fundamental operations and characteristics. Athira et al. [27] proposed an entropy measurement for PFSS and distance metrics, which were later utilized to address decision-making difficulties in [28]. These developments have markedly improved the applicability of SS-based methodologies in decision-making contexts.

Smarandache [29] proposed the neutrosophic set (NS) idea to tackle incomplete information by incorporating MD, NMD, and indeterminacy. Zulqarnain et al. [30] advanced this concept by introducing the generalized neutrosophic TOPSIS technique, which they utilized for supplier selection. Nevertheless, these models are incapable of adequately addressing split parameters. To address this issue, Smarandache [31] proposed hypersoft sets (HSS) by converting the single-parametric function f into a multi-parameter (sub-attribute) function. Smarandache asserts that HSS has superior capabilities for managing uncertainty compared to conventional SS. Since then, several scholars have investigated and expanded HSS in diverse manners [32-33].

Current methodologies concentrate exclusively on the MD and NMD values of sub-attributes, neglecting situations where, for instance, MD=0.7 and NMD=0.6, hence resulting in the condition 0.7+0.6 $\leq$ 1 for any sub-attribute of the alternatives. These theories also fail to account for sub-attribute indeterminacy in *n*-tuple attributes. The suggested similarity measures address this issue by incorporating sub-attributes through MD and NMD values provided that $0 \leq \kappa_{\mathcal{F}(\check{d}_k)}(t) + \delta_{\mathcal{F}(\check{d}_k)}(t) \leq 1$. This method guarantees effective management of ambiguity and offers a dependable structure for decision-making. Furthermore, the verbal similarity component of the proposed strategy augments its accountability and efficacy, rendering it a more advantageous option for such issues.

The following research is organized: In section 2, we recollected some basic definitions used in the subsequent sequel, such as SS, HSS, IFHSS, and interval-valued intuitionistic fuzzy hypersoft sets (IVIFHSS). Section 3 proposes similarity measures, such as cosine and set-theoretics, for IFHSS's properties. Using our developed similarity measures, we established a decision-making technique to solve decision-making complications. In Section 4, we use the proposed similarity measures for decision-making. Section 5 provides a brief comparative analysis of the proposed technique with current methods. Finally, Section 6 outlines the results and future directions.

2. Preliminaries

This section reviews basic definitions, such as soft sets, hypersoft sets, IFHSS, and IVIFHSS, that help construct the following manuscript structure.

Definition 1 [18]. Let U signify the universe of discourse, and $\mathbb{N}$ represent the collection of qualities. Let $\mathcal{P}(U)$ denote the power set of U, and let A be a subset of $\mathbb{N}$. A pair (Ω, A) is defined as a S.S. over U, with its mapping articulated as:

$$\Omega: \boldsymbol{A} \to \boldsymbol{\mathcal{P}}(\boldsymbol{U}). \tag{1}$$

Also, it can be defined as follows:

$$(\Omega, \boldsymbol{A}) = \{\Omega(\boldsymbol{t}) \in \boldsymbol{\mathcal{P}}(\boldsymbol{U}): \boldsymbol{t} \in \mathbb{N}, \Omega(\boldsymbol{t}) = \emptyset \textbf{ if } \boldsymbol{t} \notin \boldsymbol{A}\}. \tag{2}$$

Definition 2 [31]. Let U denote a universe of discourse and let $\mathcal{P}(U)$ signify its power set. Let t = $\{t_1, t_2, t_3,\ldots,t_n\}$ (n ≥ 1) with T_i idenote a collection of attributes and their associated sub-attributes, such that $T_i \cap T_j = \phi$ for $i \neq j$ and $i, j \in \{1,2,\ldots,n\}$. Let $T_1 \times T_2 \times T_3 \times \ldots \times T_n = \ddot{A} = \{d_{1h} \times d_{2k} \times \cdots \times d_{nl}\}$, where $1 \leq h \leq \alpha$, $1 \leq k \leq \beta$, and $1 \leq l \leq \gamma$, and $\alpha, \beta, \gamma \in$ N. The pair $(\mathcal{F}, T_1 \times T_2 \times T_3 \times \ldots \times T_n) = (\Omega, \ddot{A})$ is defined as:

$$\Omega: \boldsymbol{T_1} \times \boldsymbol{T_2} \times \boldsymbol{T_3} \times \ldots \times \boldsymbol{T_n} = \ddot{\boldsymbol{A}} \to \boldsymbol{\mathcal{P}}(\boldsymbol{U}). \tag{3}$$

It is also defined as:

$$(\Omega, \ddot{A}) = \{\breve{\boldsymbol{d}}, \Omega_{\ddot{\boldsymbol{A}}}(\breve{\boldsymbol{d}}): \breve{\boldsymbol{d}} \in \ddot{\boldsymbol{A}},\ \Omega_{\ddot{\boldsymbol{A}}}(\breve{\boldsymbol{d}}) \in \boldsymbol{\mathcal{P}}(\boldsymbol{U})\}. \tag{4}$$

Definition 3 [36]. U be a universe of discourse, and $\mathbb{N}$ be a set of attributes. Then, a pair $(\Omega, \mathbb{N})$ is called an interval-valued intuitionistic fuzzy soft set (IVIFSS) over U. Its mapping can be expressed as:

$$\Omega: \mathbb{N} \to \boldsymbol{IK^U}, \tag{5}$$

where IK^U represents the collection of interval-valued intuitionistic fuzzy subsets of the universe of discourse U. Also:

$$(\Omega, \mathbb{N}) = \{\boldsymbol{x}, ([\boldsymbol{\kappa}_A^l(\boldsymbol{t}), \boldsymbol{\kappa}_A^u(\boldsymbol{t})], [\boldsymbol{\delta}_A^l(\boldsymbol{t}), \boldsymbol{\delta}_A^u(\boldsymbol{t})]) | \boldsymbol{t} \in \boldsymbol{A}\}, \tag{6}$$

where $[\kappa_A^l(t), \kappa_A^u(t)]$ and $[\delta_A^l(t), \delta_A^u(t)]$ represent the MD and NMD intervals, respectively. Also, $\kappa_A^l(t), \kappa_A^u(t), \delta_A^l(t), \delta_A^u(t) \in [0,1]$ as well as satisfy the subsequent condition $0 \leq \kappa_A^u(t) + \delta_A^u(t) \leq 1$, with $A \subset \mathbb{N}$.

Definition 4 [35]. Let U denotes a universe of discourse and let $\mathcal{P}(U)$ signifies its power set. Let $t = \{t_1, t_2, t_3, \ldots, t_n\}$ ($n \geq 1$) with T_i idenote a collection of attributes and their associated sub-attributes such that $T_i \cap T_j = \phi$ for $i \neq j$ and $i, j \in \{1,2,\ldots,n\}$. Let $T_1 \times T_2 \times T_3 \times \ldots \times T_n = \ddot{A} = \{d_{1h} \times d_{2k} \times \cdots \times d_{nl}\}$, where $1 \leq h \leq \alpha$, $1 \leq k \leq \beta$, $1 \leq l \leq \gamma$, and $\alpha, \beta, \gamma \in$ N. Then, the pair ($\mathcal{F}$, $T_1 \times T_2 \times T_3 \times \ldots \times T_n$) = ($\Omega$, $\ddot{A}$) is known as IVIFHSS and is defined as follows:

$$\Omega\text{:}\ \boldsymbol{T_1} \times \boldsymbol{T_2} \times \boldsymbol{T_3} \times \ldots \times \boldsymbol{T_n} = \ddot{\boldsymbol{A}} \rightarrow \boldsymbol{IVIFHS^U}. \tag{7}$$

It is also defined as:

$$(\Omega, \ddot{\boldsymbol{A}}) = \left\{\left(\breve{\boldsymbol{d}}, \Omega_{\ddot{\boldsymbol{A}}}(\breve{\boldsymbol{d}})\right) : \breve{\boldsymbol{d}} \in \ddot{\boldsymbol{A}},\ \Omega_{\ddot{\boldsymbol{A}}}(\breve{\boldsymbol{d}}) \in \boldsymbol{IVPFS^U} \in [\boldsymbol{0}, \boldsymbol{1}]\right\}, \tag{8}$$

where $\Omega_{\ddot{A}}(\breve{d}) = \left\{\langle \zeta, \kappa_{\Omega(\breve{d})}(\zeta), \delta_{\Omega(\breve{d})}(\zeta) \rangle : \zeta \in U\right\}$. Also, $\kappa_{\Omega(\breve{d})}(\zeta) = \left[\kappa^l_{\Omega(\breve{d})}(\zeta), \kappa^u_{\Omega(\breve{d})}(\zeta)\right]$ and $\delta_{\Omega(\breve{d})}(\zeta) = \left[\delta^l_{\Omega(\breve{d})}(\zeta), \delta^u_{\Omega(\breve{d})}(\zeta)\right]$, where $\kappa_{\Omega(\breve{d})}(\zeta)$ and $\delta_{\Omega(\breve{d})}(\zeta)$ represents the membership and non-membership intervals, respectively, such as $\kappa^l_{\Omega(\breve{d})}(\zeta), \kappa^u_{\Omega(\breve{d})}(\zeta)$, $\delta^l_{\Omega(\breve{d})}(\zeta), \delta^u_{\Omega(\breve{d})}(\zeta) \in [0,1]$, and $0 \leq \kappa^u_{\Omega(\breve{d})}(\zeta) + \delta^u_{\Omega(\breve{d})}(\zeta) \leq 1$.

The interval-valued intuitionistic fuzzy hypersoft number (IVIFHSN) can be stated as $\mathcal{F} = \left(\left[\kappa^l_{\Omega(\breve{d})}(\zeta), \kappa^u_{\Omega(\breve{d})}(\zeta)\right], \left[\delta^l_{\Omega(\breve{d})}(\zeta), \delta^u_{\Omega(\breve{d})}(\zeta)\right]\right)$. To compute the alternative ranking, the score function and accuracy function for IVIFHSS can be stated as:

$$\boldsymbol{S}(\boldsymbol{\mathcal{F}}) = \frac{\boldsymbol{\kappa}^{\boldsymbol{l}}_{\Omega(\breve{d})}(\boldsymbol{\zeta}) + \boldsymbol{\kappa}^{\boldsymbol{u}}_{\Omega(\breve{d})}(\boldsymbol{\zeta}) + \boldsymbol{\delta}^{\boldsymbol{l}}_{\Omega(\breve{d})}(\boldsymbol{\zeta}) + \boldsymbol{\delta}^{\boldsymbol{u}}_{\Omega(\breve{d})}(\boldsymbol{\zeta})}{\boldsymbol{4}}, \tag{9}$$

and

$$\boldsymbol{A}(\boldsymbol{\mathcal{F}}) = \frac{\left(\boldsymbol{\kappa}^{\boldsymbol{l}}_{\Omega(\breve{d})}(\boldsymbol{\zeta})\right)^{\boldsymbol{2}} + \left(\boldsymbol{\kappa}^{\boldsymbol{u}}_{\Omega(\breve{d})}(\boldsymbol{\zeta})\right)^{\boldsymbol{2}} + \left(\boldsymbol{\delta}^{\boldsymbol{l}}_{\Omega(\breve{d})}(\boldsymbol{\zeta})\right)^{\boldsymbol{2}} + \left(\boldsymbol{\delta}^{\boldsymbol{u}}_{\Omega(\breve{d})}(\boldsymbol{\zeta})\right)^{\boldsymbol{2}}}{\boldsymbol{2}}. \tag{10}$$

3. Similarity Measure for Interval-Valued Intuitionistic Fuzzy Hypersoft Sets

Over the years, many mathematicians have developed different methods to solve MCDM problems (such as AOs and decision applications) for different mixed structures. The following sections develop a cosine metric for IVIFHSS and set the theoretical similarity.

Definition 5. Let $(\mathcal{F}, \ddot{A})$ = $\left\{\left(\delta_i, \left[\kappa^l_{\mathcal{F}(\breve{d})}(\delta_i), \kappa^u_{\mathcal{F}(\breve{d})}(\delta_i)\right], \left[\delta^l_{\mathcal{F}(\breve{d})}(\delta_i), \delta^u_{\mathcal{F}(\breve{d})}(\delta_i)\right]\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G}, \dddot{A})$ = $\left\{\left(\delta_i, \left[\kappa^l_{\mathcal{G}(\breve{d})}(\delta_i), \kappa^u_{\mathcal{G}(\breve{d})}(\delta_i)\right], \left[\delta^l_{\mathcal{G}(\breve{d})}(\delta_i), \delta^u_{\mathcal{G}(\breve{d})}(\delta_i)\right]\right) \mid \delta_i \in \mathcal{U}\right\}$ be two IVIFHSSs defined over a universe of discourse $\mathcal{U}$. Then, then the similarity measure of $(\mathcal{F},\ddot{A})$ and $(\mathcal{G},\dddot{\Lambda})$ can be described as follows:

$$\mathcal{S}^1_{IVIFHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{\Lambda}))= \frac{1}{mn}\sum_{k=1}^{m}\sum_{i=1}^{n}\frac{\left(\left(\kappa^l_{\mathcal{F}(\breve{d})}(\delta_i)+\kappa^u_{\mathcal{F}(\breve{d})}(\delta_i)\right)\left(\kappa^l_{\mathcal{G}(\breve{d})}(\delta_i)+\kappa^u_{\mathcal{G}(\breve{d})}(\delta_i)\right)+\left(\delta^l_{\mathcal{F}(\breve{d})}(\delta_i)+\delta^u_{\mathcal{F}(\breve{d})}(\delta_i)\right)\left(\delta^l_{\mathcal{G}(\breve{d})}(\delta_i)+\delta^u_{\mathcal{G}(\breve{d})}(\delta_i)\right)\right)}{\left(\sqrt{\left(\left(\kappa^l_{\mathcal{F}(\breve{d})}(\delta_i)\right)^2+\left(\kappa^u_{\mathcal{F}(\breve{d})}(\delta_i)\right)^2+\left(\kappa^l_{\mathcal{G}(\breve{d})}(\delta_i)\right)^2+\left(\kappa^u_{\mathcal{G}(\breve{d})}(\delta_i)\right)^2\right)}\sqrt{\left(\left(\delta^l_{\mathcal{F}(\breve{d})}(\delta_i)\right)^2+\left(\delta^u_{\mathcal{F}(\breve{d})}(\delta_i)\right)^2+\left(\delta^l_{\mathcal{G}(\breve{d})}(\delta_i)\right)^2+\left(\delta^u_{\mathcal{G}(\breve{d})}(\delta_i)\right)^2\right)}\right)}. \quad (11)$$

Proposition 1. Let $(\mathcal{F},\ddot{A})$, $(\mathcal{G},\dddot{\Lambda})$, and $(\mathcal{H},\ddot{C}) \in$ IVIFHSS, then the following properties hold:

i. $0 \leq \mathcal{S}^1_{IVIFHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{\Lambda})) \leq 1$.
ii. $\mathcal{S}^1_{IVIFHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{\Lambda})) = \mathcal{S}^1_{IVIFHSS}((\mathcal{G},\dddot{\Lambda}),(\mathcal{F},\ddot{A}))$.
iii. If $(\mathcal{F},\ddot{A}) \subseteq (\mathcal{G},\dddot{\Lambda}) \subseteq (\mathcal{H},\ddot{C})$, then $\mathcal{S}^1_{IVIFHSS}((\mathcal{F},\ddot{A}),(\mathcal{H},\ddot{C})) \leq \mathcal{S}^1_{IVIFHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{\Lambda}))$ and $\mathcal{S}^1_{IVIFHSS}((\mathcal{F},\ddot{A}),(\mathcal{H},\ddot{C})) \leq \mathcal{S}^1_{IVIFHSS}((\mathcal{G},\dddot{\Lambda}),(\mathcal{H},\ddot{C}))$.

Proof: It is straightforward.

4. Application of the Similarity Measure for Decision-Making under IVIFHSS Environment

This part implements the established similarity measure-based methodology for decision-making.

4.1. Algorithm for the Similarity Measure of IVIFHSS

Step 1. *Choose the set of attributes*.
Step 2. *Compute the decision matrices for each alternative in expert opinion in IVIFHSNs*.
Step 3. *Compute the similarity measure by using Definition 5*.
Step 4. *The number of alternatives with the most significant value of the similarity measure is the largest value is the most suitable alternative*.
Step 5. *Compute the ranking of alternatives*.

4.2. Problem Description

An architecture corporation offers an opening for an engineering professional to manage workforce activities. Many people applied for employment. However, only four architects received invites for interviews based on their credentials and expertise. Let S = {S_1, S_2, S_3, S_4} indicate the collection of engineers chosen for the interview. To guarantee an equitable and comprehensive assessment, a managing director designates a selection panel consisting of four specialists, represented as $\mathcal{U} = \{\kappa_1,\kappa_2,\kappa_3,\kappa_4\}$. The team of experts outlines a set of criteria to evaluate applicants for the civil engineer position, described as $\mathfrak{L}$={$\ell_1 = \text{experiance}, \ell_2 = \text{dealing skills}, \ell_3 = \text{qualification}$}. These characteristics involve: ℓ_1={$a_{11} \geq 20, a_{12} < 20$}, ℓ_2={$a_{21} = \text{public dealing}, a_{22} = \text{ staff dealing}$}, and ℓ_3={$a_{31} = \text{PhD in construction engineering}, a_{32} = \text{masters in construction engineering}$}.
The entire collection of sub-attributes is expressed as $\mathfrak{L}' = \ell_1 \times \ell_2 \times \ell_3$, integrating all feasible permutations of these values. The specialists examine the applicants based on specified standards and submit a comprehensive evaluation statement to the organization's executive director. Following

this assessment, the executive in charge renders the final judgment by choosing the most appropriate applicant. Let $\mathfrak{L}'$ = $\ell_1 \times \ell_2 \times \ell_3$ = $\{a_{11}, a_{12}\} \times \{a_{21}, a_{22}\} \times \{a_{31}, a_{32}\}$ be a set of sub-attributes as:

$$\begin{Bmatrix} (a_{11}, a_{21}, a_{31}), (a_{11}, a_{21}, a_{32}), (a_{11}, a_{22}, a_{31}), (a_{11}, a_{22}, a_{32}), \\ (a_{12}, a_{21}, a_{31}), (a_{12}, a_{21}, a_{32}), (a_{12}, a_{22}, a_{31}), (a_{12}, a_{22}, a_{32}) \end{Bmatrix}.$$

Therefore, $\mathfrak{L}'$ = $\{\breve{\alpha}_1, \breve{\alpha}_2, \breve{\alpha}_3, \breve{\alpha}_4, \breve{\alpha}_5, \breve{\alpha}_6, \breve{\alpha}_7, \breve{\alpha}_8\}$. Every expert will classify the options according to these sub-attributes, employing an IVIFHSN technique. This process guarantees a thorough evaluation of every possibility, enabling the hiring of the best applicant according to the established criteria.

4.3. Implementation of IVIFHSS for Decision-Making

The construction company developed hiring standards for the civil engineer job, assessing all shortlisted applicants (S) according to specified sub-attributes utilizing an IVIFHSN methodology, as outlined in Table 1.

Table 1
Decision matrix of related organization

S	$\breve{\alpha}_1$	$\breve{\alpha}_2$	$\breve{\alpha}_3$	$\breve{\alpha}_4$
κ_1	([.3, .5], [.2, .4])	([.2, .3], [.5, .7])	([.1, .3], [.4, .6])	([.3, .5], [.3, .6])
κ_2	([.5, .6], [.1, .3])	([.5, .7], [.1, .2])	([.3, .4], [.2, .5])	([.1, .2], [.3, .5])
κ_3	([.2, .4], [.5, .6])	([.2, .4], [.3, .4])	([.2, .3], [.1, .4])	([.2, .3], [.1, .6])
κ_4	([.2, .3], [.5, .7])	([.3, .4], [.2, .5])	([.3, .5], [.3, .6])	([.1, .2], [.4, .6])
S	$\breve{\alpha}_5$	$\breve{\alpha}_6$	$\breve{\alpha}_7$	$\breve{\alpha}_8$
κ_1	([.2, .4], [.4, .5])	([.4, .6], [.2, .4])	([.2, .3], [.3, .4])	([.4, .5], [.1, .2])
κ_2	([.4, .5], [.3, .5])	([.3, .5], [.1, .3])	([.1, .2], [.2, .4])	([.1, .2], [.5, .6])
κ_3	([.2, .5], [.5, .5])	([.3, .5], [.1, .3])	([.2, .4], [.1, .2])	([.1, .2], [.2, .3])
κ_4	([.4, .6], [.1, .3])	([.1, .2], [.2, .4])	([.2, .4], [.2, .5])	([.1, .3], [.4, .6])

The input from the team of experts is provided in Tables 2-5.

Table 2
Expert's opinion on the alternative S_1

S	$\breve{\alpha}_1$	$\breve{\alpha}_2$	$\breve{\alpha}_3$	$\breve{\alpha}_4$
κ_1	([.2, .4], [.4, .5])	([.3, .4], [.2, .5])	([.2, .3], [.1, .2])	([.2, .4], [.4, .6])
κ_2	([.3, .4], [.2, .5])	([.4, .7], [.1, .2])	([.4, .5], [.1, .2])	([.5, .7], [.1, .2])
κ_3	([.5, .6], [.2, .3])	([.1, .2], [.2, .4])	([.7, .8], [.1, .2])	([.1, .3], [.1, .5])
κ_4	([.3, .5], [.3, .4])	([.2, .4], [.2, .5])	([.2, .4], [.1, .2])	([.4, .7], [.1, .2])
S	$\breve{\alpha}_5$	$\breve{\alpha}_6$	$\breve{\alpha}_7$	$\breve{\alpha}_8$
κ_1	([.6, .7], [.2, .3])	([.2, .5], [.2, .3])	([.1, .2], [.2, .3])	([.4, .6], [.2, .3])
κ_2	([.1, .2], [.1, .5])	([.2, .4], [.1, .2])	([.2, .4], [.3, .6])	([.3, .4], [.2, .4])
κ_3	([.1, .4], [.1, .2])	([.2, .5], [.2, .4])	([.3, .5], [.2, .4])	([.5, .7], [.1, .2])
κ_4	([.5, .6], [.2, .3])	([.2, .4], [.3, .5])	([.4, .6], [.2, .3])	([.1, .5], [.2, .5])

Table 3
Expert's opinion on the alternative S_2

S	$\breve{a}_1$	$\breve{a}_2$	$\breve{a}_3$	$\breve{a}_4$
κ_1	([.2, .4], [.4, .6])	([.2, .3], [.3, .5])	([.1, .2], [.2, .5])	([.4, .5], [.1, .2])
κ_2	([.4, .5], [.1, .2])	([.5, .7], [.1, .2])	([.1, .3], [.2, .6])	([.2, .5], [.4, .5])
κ_3	([.3, .4], [.2, .6])	([. 2, .4], [. 2, .5])	([.2, .5], [.1, .2])	([.2, .5], [.4, .5])
κ_4	([.2, .4], [.4, .5])	([.3, .5], [.3, .5])	([.2, .5], [.4, .6])	([.2, .4], [.1, .2])
S	$\breve{a}_5$	$\breve{a}_6$	$\breve{a}_7$	$\breve{a}_8$
κ_1	([.2, .3], [.3, .5])	([.1, .2], [.2, .5])	([.1, .2], [.2, .3])	([.1, .3], [.2, .5])
κ_2	([.1, .4], [.2, .4])	([.1, .2], [.4, .6])	([.1, .4], [.4, .6])	([.1, .4], [.4, .6])
κ_3	([.3, .5], [.3, .5])	([.3, .5], [.3, .5])	([.1, .2], [.2, .5])	([.5, .7], [.1, .2])
κ_4	([.4, .5], [.1, .2])	([.1, .2], [.4, .6])	([.4, .5], [.1, .2])	([.1, .2], [.4, .6])

Table 4
Expert's opinion on the alternative S_3

S	$\breve{a}_1$	$\breve{a}_2$	$\breve{a}_3$	$\breve{a}_4$
κ_1	([.3, .4], [.4, .5])	([.3, .4], [.2, .5])	([.2, .7], [.1, .2])	([.1, .4], [.3, .6])
κ_2	([.3, .4], [.4, .5])	([.2, .7], [.1, .3])	([.4, .5], [.1, .4])	([.2, .6], [.1, .2])
κ_3	([.5, .6], [.2, .3])	([.1, .6], [.2, .4])	([.3, .8], [.1, .2])	([.1, .4], [.1, .5])
κ_4	([.3, .5], [.2, .4])	([.2, .4], [.1, .5])	([.2, .4], [.1, .5])	([.4, .7], [.1, .2])
S	$\breve{a}_5$	$\breve{a}_6$	$\breve{a}_7$	$\breve{a}_8$
κ_1	([.2, .7], [.2, .3])	([.2, .5], [.2, .3])	([.1, .2], [.2, .3])	([.4, .6], [.2, .3])
κ_2	([.3, .4], [.1, .5])	([.2, .4], [.1, .6])	([.2, .4], [.3, .6])	([.3, .4], [.2, .4])
κ_3	([.1, .4], [.1, .5])	([.2, .5], [.2, .4])	([.3, .5], [.2, .4])	([.5, .7], [.1, .2])
κ_4	([.5, .6], [.2, .3])	([.2, .4], [.3, .5])	([.4, .6], [.2, .3])	([.1, .5], [.2, .5])

Table 5
Expert's opinion on the alternative S_4

S	$\breve{a}_1$	$\breve{a}_2$	$\breve{a}_3$	$\breve{a}_4$
κ_1	([.3, .4], [.4, .6])	([.2, .3], [.3, .5])	([.1, .4], [.2, .5])	([.4, .5], [.1, .3])
κ_2	([.4, .5], [.1, .2])	([.5, .7], [.1, .2])	([.1, .3], [.2, .6])	([.2, .5], [.4, .5])
κ_3	([.3, .4], [.4, .6])	([. 3, .4], [. 2, .5])	([.2, .5], [.1, .4])	([.2, .5], [.4, .5])
κ_4	([.2, .4], [.4, .5])	([.3, .5], [.3, .5])	([.3, .5], [.4, .6])	([.2, .4], [.1, .5])
S	$\breve{a}_5$	$\breve{a}_6$	$\breve{a}_7$	$\breve{a}_8$
κ_1	([.2, .4], [.3, .5])	([.1, .4], [.2, .5])	([.2, .5], [.2, .3])	([.2, .3], [.2, .5])
κ_2	([.3, .4], [.2, .4])	([.1, .3], [.4, .6])	([.1, .4], [.4, .6])	([.3, .4], [.4, .6])
κ_3	([.3, .5], [.3, .5])	([.3, .5], [.3, .5])	([.1, .4], [.2, .5])	([.5, .7], [.1, .2])
κ_4	([.4, .5], [.1, .4])	([.1, .3], [.4, .6])	([.4, .5], [.1, .4])	([.1, .2], [.4, .6])

The similarity measure was computed by using Definition 5 and Tables 1-5. As a result, by using Equation (11) we got $\mathcal{S}^1_{IVIFHSS}(S,S_1)$ = 0.07007. Also, cosine similarity measures can be found between $\mathcal{S}^1_{IVIFHSS}(S,S_2)$, $\mathcal{S}^1_{IVIFHSS}(S,S_3)$, and $\mathcal{S}^1_{IVIFHSS}(S,S_4)$ as $\mathcal{S}^1_{IVIFHSS}(S,S_2)$ = 0.06771, $\mathcal{S}^1_{IVIFHSS}(S,S_3)$ = 0.06943, and $\mathcal{S}^1_{IVIFHSS}(S,S_4)$ = 0.06874. This shows that $\mathcal{S}^1_{IVIFHSS}(S,S_1) > \mathcal{S}^1_{IVIFHSS}(S,S_3) > \mathcal{S}^1_{IVIFHSS}(S,S_4) > \mathcal{S}^1_{IVIFHSS}(S,S_2)$.

According to the assessment and ranking, the alternative S_1 is recognized as the most appropriate and precisely linked with the parameters. The conclusive ranking of the alternatives is $S_1 > S_3 > S_4 > S_2$.

5. Discussion and Comparative Analysis

5.1. Superiority of the Proposed Approach

Although many measures are available for evaluating alternative methodologies, the suggested method is a significant distinction (Table 6). Current hybrid structures, such as FS, IFS, IVFS, IVIFS, and IVIFSS, cannot provide exhaustive insights regarding the circumstances. The suggested IVIFHSS adeptly handles various sub-attributes of both M. and NMD, rendering it highly appropriate for decision-making operations. The similarity measures linked to conventional mixed structures are insufficient compared to the strengths of IVIFHSS. The comparison examination of previous techniques, as illustrated in Table 6, underscores this benefit.

The suggested approach exhibits greater potential than alternatives grounded in intuitionistic fuzzy sets, effectively managing complexity with markedly enhanced efficiency. The similarity measure developed for IVIFHSS exceeds the efficacy of existing measures, underscoring its superiority for decision-making procedures.

Table 6
Comparison between IVIFHSS and some existing techniques

	Set	Truthiness	Falsity	Param.	Attributes	Aggregated information in interval form
Zadeh [1]	FS	✓	×	×	✓	×
Atanassov [2]	IFS	✓	✓	×	✓	×
Maji et al. [21]	FSS	✓	×	✓	✓	×
Maji et al. [22]	IFSS	✓	✓	✓	✓	×
Zulqarnain et al. [33]	IFHSS	✓	✓	✓	✓	×
Atanassov [34]	IVIFS	✓	✓	×	✓	✓
Jiang et al. [35]	IVIFSS	✓	✓	✓	✓	✓
Turksen [37]	IVFS	✓	×	×	✓	✓
Proposed approach	*IVIFHSS*	✓	✓	✓	✓	✓

This research concludes that the outcomes obtained from the proposed approach seem more extensive and flexible than those from existing methodologies. The strategy utilized here integrates supplementary information that significantly mitigates indecision, setting it apart from conventional decision-making methods. Furthermore, incorporating diverse hybrid configurations of FS under the paradigm of IVIFHSS augments its adaptability since it encompasses certain circumstances designed for enhanced relevance. This method facilitates an accurate and logical depiction of general details about objects, as outlined in Table 6. Thus, the suggested strategy is especially adept at managing imperfect and uncertain data in decision-making.

5.2. Discussion

This section presents a novel algorithmic framework inside the IVIFHSS methodology, employing sophisticated similarity measures. The suggested algorithm is implemented to address the realistic issue of finding the most appropriate applicant for a civil engineer position within a corporation. The findings confirm the algorithm's efficacy and applicability, underscoring its significance in decision-making processes. This assessment indicates that $S1$ is the most suitable candidate for the civil engineer position. Comparison with alternative methods demonstrates that the recommended strategy constantly surpasses them. This study's outcomes demonstrate that the proposed approach produces better outcomes than conventional methods. This is mainly because the constructed similarity measures proficiently manage complex and unclear data, outperforming existing methodologies.

The suggested method's principal benefit is its ability to integrate supplementary information into the data analysis process, a feat unattainable with current decision-making methodologies. It is an effective instrument for rectifying mistakes and imprecisions in decision-making operations. Moreover, the proposed methodology offers protection against conclusions derived from flawed or insufficient reasoning, hence providing more dependable and informed decision-making. This method's proven advantages and improved similarity measurements substantially advance over traditional strategies in handling complicated and unpredictable data.

6. Conclusion

This study examined fundamental ideas, particularly SS, HSS, IFHSS, and IVIFHSS. We devised a similarity measure designed for IVIFHSS and delineated its principal characteristics. Furthermore, we introduced an innovative decision-making methodology based on these methodologies. To illustrate the effectiveness of our methodology, a comprehensive example was presented to tackle a real-world decision-making issue. A comparison analysis was performed to assess the efficacy of the proposed strategies relative to existing methodologies.

IVIFHSS will be expanded to encompass polar interval-valued intuitionistic fuzzy hypersoft sets (PIVIFHSS), facilitating its application to intricate real-world challenges, including the architecture industry and sophisticated decision-making contexts. Subsequent research will create supplementary operators within the multi-PIVIFHSS framework to augment its applicability for decision-making difficulties. Additionally, alternative structural notions, including topological, algebraic, and sequential frameworks, may be examined within this paradigm, facilitating additional progress in theoretical and applied decision-making approaches.

Conflicts of Interest

The authors declare no potential conflicts of interest in relation to this study.

References

[1] Zadeh, L.A. (1965). Fuzzy sets. *Information and Control,* 8, 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X.

[2] Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 20*, 87-96.

[3] Yager, R.R. (2013). Pythagorean fuzzy subsets. In *2013 Joint IFSA World Congress and NAFIPS Annual Meeting* (IFSA/NAFIPS), Edmonton, Canada (pp. 57-61). IEEE. https://doi.org/10.1109/IFSA-NAFIPS.2013.6608375.

[4] Yager, R.R. (2013). Pythagorean membership grades in multicriteria decision making. *IEEE Transactions on Fuzzy Systems, 22*(4), 958-965. https://doi.org/10.1109/TFUZZ.2013.2278989.

[5] Zhang, X., & Xu, Z. (2014). Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets. *International Journal of Intelligent Systems, 29*(12), 1061-1078. https://doi.org/10.1002/int.21676.

[6] Wei, G., & Lu, M. (2018). Pythagorean fuzzy power aggregation operators in multiple attribute decision making. *International Journal of Intelligent Systems, 33*(1), 169-186. https://doi.org/10.1002/int.21946.

[7] Zhang, S., Hou, Y., Zhang, S., & Zhang, M. (2017). Fuzzy control model and simulation for nonlinear supply chain system with lead times. *Complexity, 2017*(1), 2017634. https://doi.org/10.1155/2017/2017634.

[8] Zhang, S., Zhang, C., Zhang, S., & Zhang, M. (2018). Discrete switched model and fuzzy robust control of dynamic supply chain network. *Complexity, 2018*(1), 3495096. https://doi.org/10.1155/2018/3495096.

[9] Zhang, S., Zhang, P., & Zhang, M. (2019). Fuzzy emergency model and robust emergency strategy of supply chain system under random supply disruptions. *Complexity, 2019*(1), 3092514. https://doi.org/10.1155/2019/3092514.

[10] Zhang, N., Qi, W., Pang, G., Cheng, J., & Shi, K. (2022). Observer-based sliding mode control for fuzzy stochastic switching systems with deception attacks. *Applied Mathematics and Computation, 427*, 127153. https://doi.org/10.1016/j.amc.2022.127153.

[11] Zhang, S., Li, S., Zhang, S., & Zhang, M. (2017). Decision of Lead-Time Compression and Stable Operation of Supply Chain. *Complexity, 2017*(1), 7436764. https://doi.org/10.1155/2017/7436764.

[12] Zhang, S., & Zhang, M. (2020). Mitigation of Bullwhip Effect in Closed-Loop Supply Chain Based on Fuzzy Robust Control Approach. *Complexity, 2020*(1), 1085870. https://doi.org/10.1155/2020/1085870.

[13] Sarwar, M., & Li, T. (2019). Fuzzy fixed point results and applications to ordinary fuzzy differential equations in complex valued metric spaces. *Hacettepe Journal of Mathematics and Statistics, 48*(6), 1712-1728. https://doi.org/10.15672/HJMS.2018.633.

[14] Peng, X., & Yang, Y. (2015). Some results for Pythagorean fuzzy sets. *International Journal of Intelligent Systems, 30*(11), 1133-1160. https://doi.org/10.1002/int.21738.

[15] Garg, H. (2019). New logarithmic operational laws and their aggregation operators for Pythagorean fuzzy set and their applications. *International Journal of Intelligent Systems, 34*(1), 82-106. https://doi.org/10.1002/int.22043.

[16] Arora, R., & Garg, H. (2019). Group decision-making method based on prioritized linguistic intuitionistic fuzzy aggregation operators and its fundamental properties. *Computational and Applied Mathematics, 38*, 36. https://doi.org/10.1007/s40314-019-0764-1.

[17] Ma, Z., & Xu, Z. (2016). Symmetric Pythagorean fuzzy weighted geometric/averaging operators and their application in multicriteria decision-making problems. *International Journal of Intelligent Systems, 31*(12), 1198-1219. https://doi.org/10.1002/int.21823.

[18] Molodtsov, D. (1999). Soft set theory—first results. *Computers & Mathematics with Applications, 37*(4-5), 19-31. https://doi.org/10.1016/S0898-1221(99)00056-5.

[19] Maji, P.K., Biswas R., & Roy, A.R. (2003). Soft set theory. *Computers and Mathematics with Applications, 45*(4–5) 555-562. https://doi.org/10.1016/S0898-1221(03)00016-6.

[20] Maji, P.K., Roy, A.R., & Biswas, R. (2002). An application of soft sets in a decision making problem. *Computers & Mathematics with Applications, 44*(8-9), 1077-1083.

[21] Maji, P.K., Biswas R., & Roy, A.R. (2001). Fuzzy soft sets. *Journal of Fuzzy Mathematics, 9*, 589-602.

[22] Maji, P.K., Biswas R., & Roy, A.R. (2001). Intuitionistic fuzzy soft sets. *Journal of Fuzzy Mathematics, 9*, 677-692.

[23] Garg, H., & Arora, R. (2018). Generalized and group-based generalized intuitionistic fuzzy soft sets with applications in decision-making. *Applied Intelligence, 48*, 343-356. https://doi.org/10.1007/s10489-017-0981-5.

[24] Xia, Y., Wang, J., Meng, B., & Chen, X. (2020). Further results on fuzzy sampled-data stabilization of chaotic nonlinear systems. *Applied Mathematics and Computation, 379*, 125225. https://doi.org/10.1016/j.amc.2020.125225.

[25] Zulqarnain, R. M., Xin, X. L., Saqlain, M., & Khan, W. A. (2021). TOPSIS Method Based on the Correlation Coefficient of Interval-Valued Intuitionistic Fuzzy Soft Sets and Aggregation Operators with Their Application in Decision-Making. *Journal of Mathematics, 2021*(1), 6656858. https://doi.org/10.1155/2021/6656858.

[26] Peng, X., Yang, Y., Song, J., & Jiang, Y. (2015). Pythagorean fuzzy soft set and its application. *Computer Engineering, 41*(7), 224-229.

[27] Athira, T. M., John, S. J., & Garg, H. (2020). A novel entropy measure of Pythagorean fuzzy soft sets. *AIMS Mathematics, 5*(2), 1050-1061.

[28] Athira, T. M., John, S. J., & Garg, H. (2019). Entropy and distance measures of Pythagorean fuzzy soft sets and their applications. *Journal of Intelligent & Fuzzy Systems, 37*(3), 4071-4084. https://doi.org/10.3233/JIFS-190217.

[29] F. Smarandache, F. (1998). Neutrosophy: neutrosophic probability, set, and logic: analytic synthesis & synthetic analysis. Rehoboth Am. Res. Press.

[30] Zulqarnain, R. M., Xin, X. L., Saeed, M., Smarandache, F., & Ahmad, N. (2020). *Generalized neutrosophic TOPSIS to solve multi-criteria decision-making problems* (Vol. 38). Infinite Study.

[31] Smarandache, F. (2018). Extension of Soft Set to Hypersoft Set, and then to Plithogenic Hypersoft Set. *Neutrosophic Sets and Systems, 22*, 168-170.

[32] Rana, S., Qayyum, M., Saeed, M., Smarandache, F., & Khan, B.A. (2019). Plithogenic Fuzzy Whole Hypersoft Set: Construction of Operators and their Application in Frequency Matrix Multi Attribute Decision Making Technique. *Neutrosophic Sets and Systems, 28*, 34-50.

[33] Zulqarnain, R.M., Xin, X.L., Saqlain, M., & Smarandache, F. (2020). Generalized Aggregate Operators on Neutrosophic Hypersoft Set. *Neutrosophic Sets and Systems, 36*, 271-281.

[34] Atanassov, K.T. (1999). Interval-valued intuitionistic fuzzy sets. In *Intuitionistic Fuzzy Sets* (pp. 139-177). Physica, Heidelberg.

[35] Jiang, Y., Tang, Y., Chen, Q., Liu, H., & Tang, J. (2010). Interval-valued intuitionistic fuzzy soft sets and their properties. *Computers & Mathematics with Applications, 60*(3), 906-918. https://doi.org/10.1016/j.camwa.2010.05.036.

[36] Debnath, S. (2022). Interval-Valued Intuitionistic Hypersoft Sets and Their Algorithmic Approach in Multi-criteria Decision Making. *Neutrosophic Sets and Systems*, *48*, 226-250.

[37] Turksen, I.B. (1986). Interval valued fuzzy sets based on normal forms. *Fuzzy Sets and Systems, 20*(2), 191-210.