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Hybrid Approach for COVID-19 Vaccine Distribution

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ABSTRACT

Since December 2019, the entire world has become fully uncontrolled due to the critical and unpredictable nature of the COVID-19 virus. The researchers have proposed various precautionary measures to protect ourselves from it. One of the most established and acceptable tools to control the COVID-19 pandemic is vaccination. Pharmaceutical companies are trying their best to supply vaccines according to requirements within a short period. It has become a challenging task to allocate a limited quantity of vaccines among the state and union territories with respect to multiple aspects. A number of factors are involved and have various impacts on the distribution of vaccines. The impacts of the vaccine distribution factors are estimated using the parameters that are responsible for spreading the COVID-19 infection, such as population density, active cases, infection rate, and total infected persons. In this article, we propose a proportional and infection-based vaccine allocation technique to distribute the vaccine among states or union territories of India based on six COVID-19-related factors to reduce the day-wise infection rate rapidly. Accuracy rate and three distribution grades are used to measure the performance of the proposed methods. Then, a hybrid method is developed by combining the proportional and infection-based vaccine allocation techniques to improve the accuracy rate of the vaccine distribution. Finally, we compare the proportional and infection-based vaccine allocation technique with the hybrid approach, where the hybrid approach performs better.

1. Introduction

The first outbreak of an unknown virus with severe acute respiratory syndrome reportedly occurred in Wuhan, China, in mid-December 2019 [1]. World Health Organization (WHO) identified the characteristics of the virus and denoted it as COVID-19 [2]. Since December 2019, most countries around the world have been badly affected by the COVID-19 virus. According to the WHO's observation, most of the people who were affected by COVID-19 recovered from the common

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treatment without hospitalization [3]. But 10%-20% of COVID-19 patients were seriously ill and faced dangerous breathing problems. Furthermore, it was reported that the infection rates of COVID-19, as well as fatality rates, were higher for elderly people who had some comorbidity conditions, such as diabetes and hypertension [4]. Unfortunately, the condition of many of the elderly patients was severe. One of the main causes of this was that elderly patients are more prone to multisystem organ dysfunction [5]. Additionally, patients with pre-existing comorbidities are at a higher risk of developing severe coronavirus infections than patients with a healthy medical history [6]. The probable treatment of COVID-19-infected patients, and the performance of various medicines are analyzed in [7].

WHO (World Health Organization) and the Alliance Organization struggled to manage the pandemic situation of COVID-19 [4]. They announced various types of instructions to take as precautions for preventing the infection COVID-19, like washing hands regularly with soap and water, maintaining social distance from people coughing or sneezing, avoiding touching their face, covering mouth and nose with a mask, etc. At the same time, they advised to avoid unnecessary political or religious gathering during this period. Primarily, this advice helped to reduce COVID-19 cases. Many of the COVID-19-affected countries took some strong decisions like a full lockdown to break the exploration chain of the COVID-19 virus. Those strong decisions helped to reduce the chance of being exposed to the virus or spreading it to others, but these are not enough to stop it permanently [8]. The world-famous virologists deeply investigated and proposed that the vaccination might be the ultimate solution. On 11th January 2020, the genetic sequence of COVID-19 was globally published, which boosted the development of the preventative COVID-19 vaccine [9]. The vaccine development was accelerated in early 2020 by the investment of ten billion dollars from the multinational pharmaceutical industries, billionaire persons, and some countries. By June 2020, various research groups throughout the world developed a dozen of the probable vaccines and considered the global vaccination program to immunize against COVID-19 infection for phase wise testing. As of 24 June 2020, China approved the CanSino vaccine for limited use in the military. Russia started their vaccination 'Sputnik V' on 11th August 2020 for emergency purposes. From late 2020, the United Kingdom, Bahrain, UAE, and the United States started the vaccination officially of various COVID-19 vaccines like BNT162b2, Pfizer–BioNTech, and BBIBP-CorV [9].

After the approval of the various COVID-19 vaccines, the issue was to distribute the COVID-19 vaccine globally. The fundamental target was to distribute the vaccine among all countries equally. At present, there are no predefined protocols to force countries to share extra vaccines if they have excessive amounts of vaccines. The Coalition for Epidemic Preparedness Innovations reported that till now there are no clear agreements for the principles of the fair allocation system. Moreover, none of the global organizations responded to take full responsibility to manufacture and supply the vaccines as per the global scale order. Matrajt *et al.,* [10] introduced a mathematical model to manage the vaccine crisis during the vaccination. The model consisted of a pair of optimization algorithms based on four various metrics. They reported that the pandemic can be reduced if the effectiveness of the vaccines is more than 50%.

In India, emergency basis COVID-19 vaccination started officially on 16 January 2021. As of 24 May 2021, India had completed 41864206 full doses of vaccination. Initially, two vaccines got the approval from the government of India for starting the vaccination in emergency use, i.e., Covishield (a brand of the Oxford–AstraZeneca vaccine manufactured by the Serum Institute of India) and Covaxin (developed by Bharat Biotech). In April 2021, Sputnik V (distributed by Dr. Reddy's Laboratories) was approved as a third vaccine, which was deployed by May 2021. Presently, Serum Institute of India and Bharat Biotech produce 840 million and 700 million doses of vaccines every year. Initially, the production of vaccines was not adequate to satisfy the demand of people. The people of our country were fully exhausted due to home quarantine for a long period. They were not allowed to move freely outside their home. They observed the effectiveness of vaccines. Most of the people from various communities showed their interest and enlisted their names for vaccination to protect themselves from the COVID-19 infection.

A number of precautions, such as wearing a mask, cleaning hands, and maintaining a safe distance, are taken to overcome the pandemic situation, which is shown in Figure 1. But sometimes, those precautions are not sufficient to stop the spreading of COVID-19 among the communities, and unable to recover the infected persons with comorbidities. Then, the COVID-19-affected countries decided to implement a full lockdown to break the exploration chain. As a side effect of implementing the lockdown, the financial structure collapsed, and common people, including daily wage workers, faced huge problems like food crises, joblessness, deriving from quality treatment, etc. In this situation, the only acceptable option to scientifically control COVID-19 was vaccination. The vaccine can create the antioxidant of the virus and boost the immunity power. After the final phase of a clinical trial on 25800 people, the efficiency of vaccination was up to 81%. According to the performance reports of vaccination, some vaccinated people faced problems like headaches, fever, body aches, rashes, etc., as the side effects [8, 11-13]. In India, vaccination was started on 18th January 2021. The performance reports of vaccines were generated by the Ministry of Health, Government of India, and showed that the success rate was up to 99.95% [14, 15]. Then, the whole community realized and understood the importance of vaccination and was keenly interested in the vaccinations. As a result, a huge crisis of vaccines was found in India due to its high population (2nd largest population). However, the world's largest vaccine manufacturing industry, namely Serum Institute of India (SII), is in Pune, Maharastra, India. The SII produced the vaccine known as Covishield [16]. The day-wise vaccine production capacity of SII was 840 million doses per annum. At the same time, another reputed vaccine manufacturing company, namely Bharat BioTech, which is located in Hyderabad, Telangana, produced 700 million doses of vaccine (COVAXIN [17]) per year. However, those large units of vaccines produced are not able to fulfill the day-to-day demands of the country. Another important task is to carry the vaccine from the manufacturer plant to the vaccine distribution point (VDP). During the transportation and storage of the vaccines, the requirement of temperature is between 2 to 8˚C [16]. The vaccine must be pushed within six hours after opening the pack to get maximum benefit. Hence, it is a challenging task to distribute the vaccines among the State and Union Territories (SUTs) in India and transport the vaccine from the manufacturing plant to the VDP. For a highly populated country like India, where the population density is higher in some parts of the country and comparatively less in other parts, the active cases, severe cases, and infection rates of COVID-19 are massive in the highly populated sections of the country whereas less populated sections are less affected. Then, the vaccine distribution among the SUTs is an important factor in reducing the number of new cases and fatalities and in proper utilization of the vaccine in due time. The vaccines should be distributed in such a way that day-wise infection rates are reduced. However, as far as we know, no such approaches have been found that can manage vaccine distribution efficiently.

To allocate the amount of vaccine for each SUT, a number of necessary factors need to be considered, which are directly or indirectly connected to the spreading of COVID-19 within the community. It is difficult to maintain the stability of vaccine distribution in highly populated countries like India since the awareness of COVID-19 vaccination has been growing rapidly among the citizens. Due to the huge rolling demand, the available vaccines are often not able to satisfy the demand. Moreover, the SUTs are located in different geographical regions, which indicates the requirement for different vaccine supply chains in different modes. Our main aim is to maintain the stability of the vaccine distribution among the SUTs. In this large-scale pandemic, our primary goal is to reduce day-wise infection in the country through vaccination among the people. In order to achieve that, initially, we propose two vaccine distribution methods: proportional-based distribution and infection-based distribution. The proportional-based vaccine distribution method distributes the vaccine among the SUTs based on the condition of parameters like the active case, infection rate, total population, and population density, whereas the infection-based distribution method distributes the vaccine based on the rate of infection, population density, average contract hours per day, and vaccination ratio. However, none of the proposed vaccine distribution methods (proportional-based and infection-based) is able to fulfill all the criteria of the SUTs. Both of the distribution methods find some difficulty in distributing the vaccine exactly among the SUTs with respect to the request for the vaccine, total population, infection rate, and active case. We observed that the vaccine distribution by those two methods in some SUTs is accurate, whereas other SUTs are either over-distributed or under-distributed. Hence, there is an obvious need to combine both of the vaccine distribution methods. Thereafter, we propose a hybrid approach by combining the proposed two approaches to balance the vaccine distribution among the SUTs. The basic objective of the hybrid vaccine distribution technique is to uniformly allocate the limited vaccine among the SUTs in accordance with their situations to reduce the day-wise infection rate. The hybrid model provides a better result than the proposed two models. We applied the hybrid approach to distribute the vaccine among the thirty-six SUTs, where it showed better performance.

The rest of the article is arranged subsequently. In Section 2, we discuss COVID-19 and its related factors. Then, in Section 3, we present the proportional, infection, and hybrid vaccine allocation methods. Next, detailed results discussion and analysis of the three vaccine allocation methods are given in Section 4. Lastly, future work and conclusions are drawn in Section 5.

2. Background

COVID-19 was unknown prior to its outbreak in Wuhan, China, in December 2019, but it is now a pandemic affecting most countries globally. On January 30, 2020, WHO declared the COVID-19 outbreak a Public Health Emergency of International Concern (PHEIC). WHO initiated the highest level of urgency to protect against the spread of COVID-19. On March 11, 2020, WHO made the assessment that COVID-19 could be characterized as a pandemic [11], and they observed that the affected persons of COVID-19 are the primary source to transmit the virus to other persons through respiratory droplets from sneezing, coughing, and talking [12]. The virus is spread primarily by aerosols, which contain dirty fomites. Recently, it was explored that COVID-19 can be transmitted from those patients who have mild to severe symptoms and also from those patients who are pre-symptomatic or asymptomatic [18]. The epidemiology of COVID-19 changed rapidly with the second wave. As of November 6, 2020, there were more than 48 million cases and 1.2 million deaths globally [13]. Sensing the critical nature of the virus, a large number of researchers throughout the world are working hard in various ways to mitigate the spread of disease, tracking the spread of disease, developing critical interventions, distributing vital medical supplies, and supporting the development of therapeutics and multiple vaccines.

There are a number of incidents and parameters that are directly or indirectly involved in the spreading of COVID-19 over the country. Those can be classified mainly into three categories such as base factors (BF), active factors (AF), and relative factors (RF), where the base factors indicate the socio-economic status of a region, active factors represent the present scenario of COVID-19 and three relative factors denote the effect of COVID-19. The base factors are the socio-economic factors of our community, like population, population density, and contract hours per day. The active factors, such as active cases, infection rate, and total infected persons, are the different quantities that represent the infection growth of COVID-19. The relative factors, such as a request for vaccination, total vaccinated persons, and fatality rate, represent the after-effects of COVID-19. Normally, six related factors, such as total population, population density, active case, total infected persons till date, vaccination till date, and request for vaccination out of the nine factors mentioned above in the three categories (BF, AF, RF) are considered for the state wise COVID-19 vaccine distributions. We have collected the information (total population, population density, active cases, total infected persons to date, vaccination till date, and request for vaccination) for all the state and union territories (SUTs) of India from authorized government agencies reports regarding COVID-19 and Census of India (CoI) of 2011 [15]. The present status of the six factors that are involved in distributing the vaccine among the SUTs is represented with the choropleth map in Figure 2. The information on those six factors helped us to calculate the required amount of vaccine needed by each of the SUTs of the country to reduce the day-wise infection rate. If the total population, population density, and total infected persons are high, then the chances of increasing the infection rate will be high. Consequently, the active case will also increase [19]. Alternatively, a high vaccination rate may reduce active as well as severe cases. The allocation of vaccines among SUTs is estimated based on the current values, significance, and max-min limit of the factors. The factors (total population, population density, active cases, total infected persons till date, vaccination till date, and request for vaccination) are related to the new infection of COVID-19 and have various impacts in our society in respect of the COVID-19 pandemic as in Table 1. The maximum and minimum values of those factors are gathered from the authorized government web portal [15], and all are positive quantities, as shown in Table 2. Presently, some vaccine distribution approaches based on disease analysis have been introduced in [20-24].

Table 1
Parameters considered for optimization

Parameters	Significance	Effects
Population	High	Increase the number of doses for vaccination Increase service points and supporting stuff for vaccination Require high capacity and speedy transportation service for carrying the vaccine
Population Density	High	Spread rapidly and explore exponentially Unable to maintain social distance Difficult to provide quality service of the affected persons
Active Cases	Increase	Vaccination is the ultimate alternative option to precaution Increased demand of vaccination Increased vaccination rate
Total Affect persons	High	The rate of exploration is high Increased the fatality rate
Vaccination still date	High	Boosting immunity Protect from severe cases Reduce the demand of vaccine
Request for vaccination	Increase	Require a high quantity of does to manage the demand Require experienced health workers to push the vaccine safely Provide sufficient resources to maintain vaccination rules Maintain COVID-19 protocol accurately pre and post-push the vaccine Manage a number of candidates in low-density region for a batch of vaccine

3. Proposed Method

Distributing the vaccine among the SUTs is a very complex task, and consistency must be maintained during the vaccine allocation [20-24]. Proper vaccine distribution, transportation, and vaccination are very important to controlling the global COVID-19 pandemic. We introduce two vaccine distribution methods, namely proportional-based vaccine allocation and infection-based vaccine allocation, for properly distributing the vaccine among the SUTs.

Table 2
Related factors of States/UTs for allocating vaccines.

Notations	Situation factors	Minimal values	Maximal values
BF_1	The total population	64473	199812341
BF_2	The population density per Km^2	17	11320
BF_3	The average contact time between individuals in the state and union territories.	5	16
AF_1	The active cases	17	50298
AF_2	The infectious rate per 10000 persons	2746	37960
AF_3	Total infected persons	770	3173261
RF_1	Request for vaccination.	3001	8949560
RF_2	The vaccination persons	10737	9338531
RF_3	The fatality rate per 10000 cases	4.7	17.8

3.1 *Proportional Based Vaccination Allocation*

The proportional vaccine allocation method is used to measure the desired amount of vaccine required to reduce the infection rate according to the six related factors of the SUTs. Those six related factors, which are strongly connected to the infection of COVID-19, are considered parameters of proportional vaccination allocation among the SUTs in this proposed method. In addition, the following variables are used to describe the proposed method.

n : The number of SUTs.
SUT_i: The ith SUT, $i = 1, 2, ..., n$.
m : The number of six related factors considered for allocating vaccines.
κ_j^i: The value of the jth factor of the ith SUT, $i = 1, 2, ..., n$; $j = 1, 2, ..., m$.
BS^* : The most desirable situation
WS_* : The least choice worst situation
ΔD: The distance between the best situation BS^* and the worst situation WS_*
D_i^* : The distance between the condition of SUTs and the best situation BS^*
SS_i: The situation of the ith SUT, $SS_i = \{\kappa_1^i, \kappa_2^i, \kappa_3^i, ..., \kappa_m^i\}, i = 1, 2, ..., n$.
RT_i : The relative similarity of the situation of SUT_i
PA_i : The allocation ratio of the available vaccines for each SUT_i
QV: The available quantity of vaccines for all SUTs.
QV_i: The allocated quantity of vaccines for ith SUT.

Some factors may have a positive impact on vaccine distribution responses, and other factors may have a negative impact. Basically, various related factors may have different dimensions. For convenience, the following remarks are considered.

Remark 1. Calculate the ratio of related factors with respect to the base point for measuring the importance of the parameters.

Remark 2. If the values of any related factors are negatively connected to the demand for vaccines, we could use inverse values of those factors in the proposed approach.

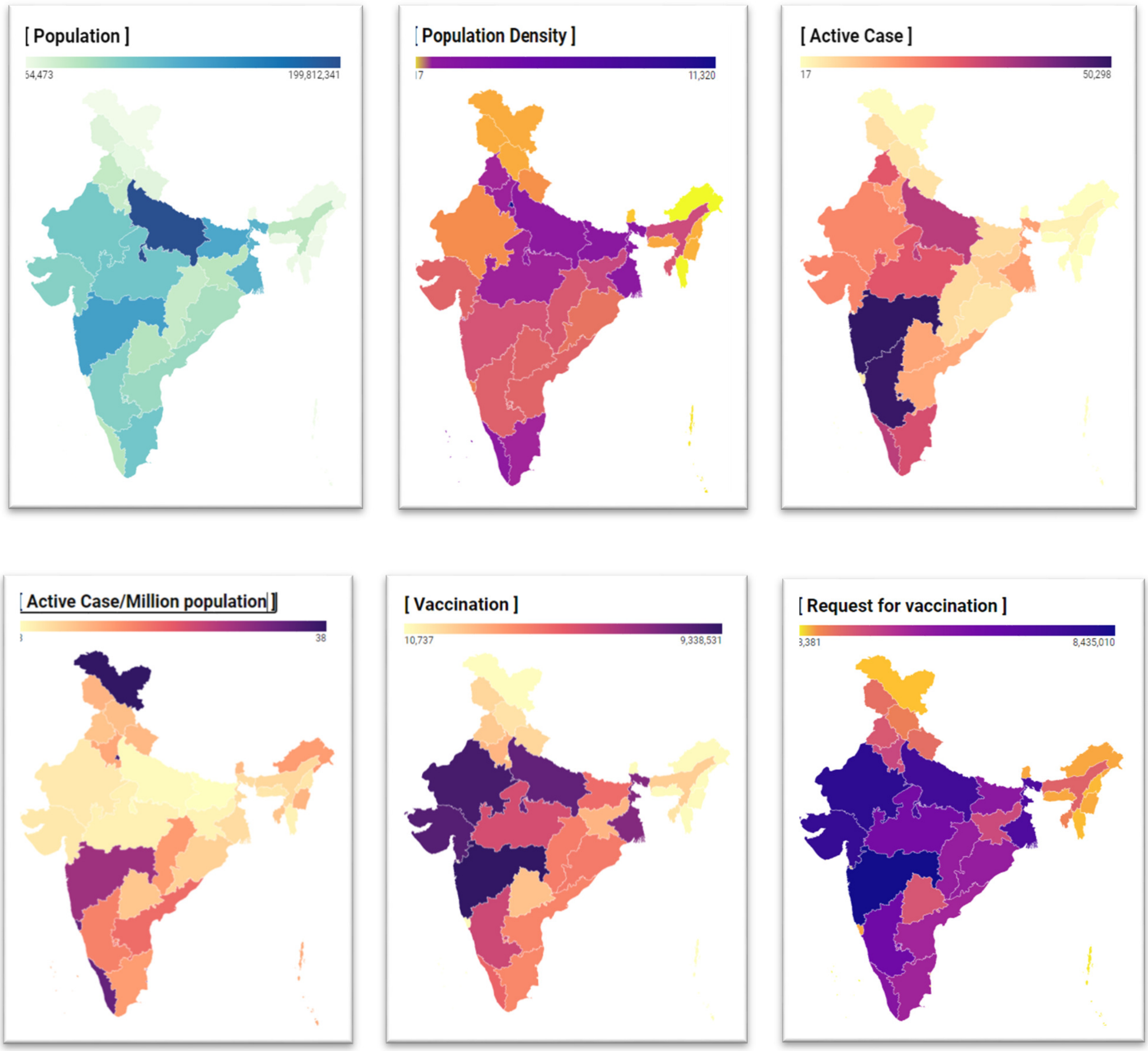

Fig. 2. State/UTs wise choropleth map of the six factors

To maintain good governance, the allocation of limited vaccines among the SUTs should be done based on their condition. If SUT_i and SUT_m are two SUTs and SUT_i demands more vaccines based on the condition of $SUT_i, 1 \leq i, m \leq n, i \neq m$, then the quantities of allocated vaccines of SUT_i should be more than SUT_m. Then, to estimate the vaccine allocation quantities, we need to measure and compare the conditions of the SUTs. The proposed mathematical model is used to measure the quantity of vaccine allocation from the present situation [25]. For this purpose, we state the following definitions.

Definition 1. The most desirable situation denoted by BS^* is made of according to the minimal value of the factors among all the SUTs, which is denoted as $BS^* = \{\kappa_1^*, \kappa_2^*, \kappa_3^*, ..., \kappa_m^*\}$ where $\kappa_j^* = \min(\kappa_j^1, \kappa_j^2, \kappa_j^3, ..., \kappa_j^n)$.

As per the definition, the desirable solutions of the function *min* () indicate the factor values showing the fewest demand for vaccines. Then, the closer condition of SUTs with respect to BS^* are more desirable. Consequently, the smallest quantities of vaccines may be allocated for the SUT.

Definition 2. The worst situation, which belongs to the least choice, is denoted by WS_*. It is made up according to the maximum factor values among all the SUTs, that is denoted as $WS_* = \{\kappa_{*1}, \kappa_{*2}, ..., \kappa_{*m}\}$ where $\kappa_{*j} = \max(\kappa_j^1, \kappa_j^2, \kappa_j^3, ..., \kappa_j^n)$.

Similarly, the solutions of the function *max* () represent that the factor values indicate the maximum demand for vaccines. So, when the SUTs are closer to WS_*, they are supposed to be in the worst situation and require the highest volume of vaccines for those SUTs.

Based on Definitions 1 and 2, the best and worst situations are respectively generated for this consequence of the minimum and maximum demand of vaccines for the SUTs. When the distance between the condition of SUT_i and the best situation BS^* is small, then the requirement of vaccine is minimal for SUT_i. Similarly, when the distance between the condition of SUT_i and the worst situation WS_* is small, then the requirement of vaccine is maximum for SUT_i.

To allocate the desired amount of vaccine, we need to develop a quantitative metric to measure the distances between the conditions of SUTs and the best or worst situations. To measure the distance, Hamming or Euclidean distance is generally considered, where Euclidean distance is found to be more beneficial than the Hamming distance metrics [26]. Meanwhile, different relative factors may have different impacts on the vaccine distribution among the SUTs, so we use weighted Euclidean distance to measure the distance D_i^* between the condition of SUTs and BS^*, and similarly, the distance ΔD between BS^* and the WS_* is measured. The weight of the factors indicates the impact of the new infection of COVID-19. This is estimated by the correlation coefficient of each factor with respect to all other factors. In order to maintain the consistency, we use the ratio of D_i^* and ΔD to represent the relative similarity of the condition of SUT_i with respect to the best situation or the worst situation according to the problem.

$$D_i^* = \sqrt{\sum_j w_{ij} \left(\kappa_j^i - \kappa_j^*\right)^2} \tag{1}$$

$$\Delta D = \sqrt{\sum_j w_{ij} \left(\kappa_{*j} - \kappa_j^*\right)^2} \tag{2}$$

Here w_{ij} denotes the relative weight of i-th SUT for the j-th factor.

The relative similarity of the situation of SUT_i is denoted by RT_i, and formulated as

$$RT_i = \frac{D_i^*}{\Delta D}. \tag{3}$$

We estimate the relative similarity of SUT_i as RT_i, with $0 \le RT_i \le 1$. If $RT_i = 0$, then the condition of SUT_i is said to be in the ideal situation. On the other hand, when $RT_i = 1$, then the condition of SUT_i indicates the worst situation. Note that the smaller RT_i indicates the small distance between the condition of SUT_i and BS* and in turn smaller amount of vaccine is demanded for SUT_i, and vice versa. Thus, the metric in Eq. (3) is effective for our vaccine allocation method and has some good properties for measuring the relative status of SUTs, as shown below.

(i) Normalization: For any SUTi; $i = 1, 2, ..., n$; it holds $0 \le RT_i \le 1$.

(ii) Monotonicity: For any two instance of SUTs, SUT_i and SUT_l, for $i, l = 1, 2, ..., n$; and $i \ne l$, if $\kappa_j^i \succ \kappa_j^l$ and other factor values are the same, then RT_i will be bigger than RT_l.

(iii) Transmissibility: For three instance of SUTs, SUT_i, SU_l and SUT_m, for $i, l, m = 1, 2, ..., n$; and $i \ne l \ne m$, if $\kappa_j^i \succ \kappa_j^l \succ \kappa_j^m$ and other factor values are the same, then RT_i will be bigger than RT_m.

Remark 3. Expression (3) reflects the relative value of the condition of SUT_i in respect to either the best situation or the worst situation. Meanwhile, in this work, we need to compare interval factor values, and Eq. (3) is used for this purpose. The comparison values of interval numbers could include both the positive and negative impacts of the related factors [27]. Expression (3) could effectively manage both instances.

The allocation ratio of the available vaccines PA_i for each of the SUTs is determined based on RT_i and defined below in Eq. (4).

$$PA_i = \frac{RT_i}{\sum_i RT_i} \tag{4}$$

Then, the allocated quantity of vaccines for the SUT_i, denoted by PQ_i, is equal to $PA_i * QV$ where QV is the available quantity of vaccines. In real-world pandemic conditions, a number of factors are directly or indirectly connected to the new infection of the COVID-19 virus within the community. All those factors do not have the same impact on spreading the virus, whereas different factors have different impacts. The impact of each factors of the SUTs are determined by the relative weight w_{ij} with respect to the conditions of all other SUTs.

Procedure

The step-by-step procedure of the proposed proportional-based vaccine allocation method is given below.

Step 1: The condition SS_i of SUTi is defined by $SS_i = \{\kappa_1^i, \kappa_2^i, \kappa_3^i, ..., \kappa_m^i\}, i = 1, 2, ..., n$, where κ_j^i be the related factor.

Step 2: Evaluate the best and worst situation BS^* and WS_* respectively from the condition of all SUTs.

Step 3: Estimate the weighted distance (D_i^*) between the condition of the SUT (SS_i) and BS^* of each SUT using (1).

Step 4: Estimate the weighted distance (ΔD) between BS^* and WS_* using Eq. (2).

Step 5: Calculate the relative similarity (RT_i) for each SUT_i using Eq. (3).

Step 6: Calculate the allocation ratio (PA_i) of each SUT using Eq. (4).

Step 7: The allocated quantity of vaccine PQ_i ($PQ_i = PA_i * QV$) for each SUT_i is computed based on the total available quantity of vaccine (QV).

Relative Weight

Relative weight analysis is a procedure to estimate the relative importance of one criterion with respect to all other criteria. There is no specific range for the values of various criteria that are related to the problem. Normally, predicting the relative importance and behavior of the criteria is impossible. The researchers cannot make actual judgments regarding the statistical significance of the various weight analyses because that procedure cannot be used to determine whether a weight is significantly different from zero. The statistical significance of a relative weight was introduced in [28]. In our study, we estimate the relative weight of each of the factors of the SUTs by measuring the distance from the factor of all SUTs, which are used for checking the individual relative importance of the factor. The relative importance of the j[th] factor of i[th] alternative are denoted as w_{ij} and $w_{ij} \geq 0$. The higher magnitude of w_{ij} indicate maximum importance of factor. The present situation of SUT is represented as $SS_i = \left\{ \kappa_1^i, \kappa_2^i, \kappa_3^i, ..., \kappa_{m1}^i \right\}, i = 1, 2, ..., n$. The following equation is used to measure the relative weight.

$$w_{ij} = \begin{cases} \dfrac{\sum_{l=1}^{m1} \left(\kappa_l^i - \kappa_l^j \right)^2}{\sum_{j=1}^{n1} \sum_{l=1}^{m1} \left(\kappa_l^i - \kappa_l^j \right)^2}, i \neq j \\ 1 \qquad\qquad i = j \end{cases} \tag{5}$$

3.2 Infection Rate Wise Vaccination Allocation

The affected persons of COVID-19 are the primary source for transferring the virus and stimulating new infection of COVID-19 within the community. The affected person may have mild to severe symptoms, and the virus is transmitted by those who are pre-symptomatic or asymptomatic [18]. The transmission is done through respiratory droplets from sneezing, coughing, and talking [12]. The authors in [29] investigated the correlation between COVID-19 infection and mortality rate with the population density. According to the investigation results, there is a connection between the mortality and new infection of COVID-19 and population density. So, factors like average contract hours per day, high population density within an area, and current active cases are directly related to the spreading of the virus among the people. Similarly, sharing, gathering, no masks, wandering, daily needs, and providing emergency services are the factors that may violate the COVID-19 protection rules, as shown in Figure 1. The vaccinated persons are safe from the new infection, whereas the unvaccinated persons have a high chance of infection. The day-wise infection rate (I) for the SUT_i is denoted by $I_i = \left(\frac{dI}{dt}\right)_i$, which is defined by the following equation:

$$\left(\frac{dI}{dt}\right)_i = C_i * D_i * AC_i + \left(1 - V_i\right) * R_i \tag{6}$$

Here C_i : The average contact hours in a day among individuals in the SUT_i.

D_i : Normalized population density/Km^2 of the SUT_i.

AC_i : Normalized active cases of the SUT_i.

V_i : Vaccination ratio with respect of the total population of the SUT_i.

R_i : Rate of infection of millions of people.

The relative infection rate of the SUT_i in respect of all SUTs is denoted as RI_i and estimated by:

$$RI_i = \frac{\sqrt{\left(I^* - I_i\right)^2}}{\sqrt{\left(I^* - I_*\right)^2}} \text{ where } I^* = \max_i \left(I_i\right) \text{ and } I_* = \min_i \left(I_i\right)$$

The vaccine allocation ratio corresponds to SUT_i is denoted by PI_i and computed as

$$PI_i = \frac{RI_i}{\sum_i RI_i} \tag{7}$$

Then, the allocated quantity of vaccine for SUT_i is denoted by IQ_i, and computed as

$$IQ_i = PI_i * QV \tag{8}$$

where QV be the available quantity of total vaccines.

Normalization

Data normalization converts the original data into a smaller or common range, such as [−1, 1] or [0.0, 1.0], and provides equal importance to the attributes. There are many methods for data normalization, such as min-max normalization, z-score normalization, normalization by decimal scaling, and neutral membership-based normalization [30]. In our proposed method, we consider the min-max normalization. Let A be a numeric attribute with *l* observed values, v_1, v_2, …, v_l, then the min-max normalization executes a linear transformation on the original data within the interval 0 and 1. Suppose that min_A and max_A are the minimum and maximum values of an attribute A. Min-max normalization converts the value v_i of A into v'_i using Eq. (9).

$$v_i^{'} = \frac{v_i - \min_A}{\max_A - \min_A} \tag{9}$$

Min-max normalization preserves the relationships among the original data values but is unable to normalize the input that belongs outside the original data range for *A*.

3.3 Hybrid Approach for Vaccine Allocation

For the accurate vaccine allocation among the SUTs, the following three basic conditions must be maintained during the vaccine allocation:

i. The sum of the allotted vaccines of each SUT must be lower or equal to the total available vaccines, i.e., $\sum_i QV_i \leq QV$.
ii. The allotted vaccines should be lower than or equal to the requested amount.
iii. Allocation of vaccines for each SUT should be at least the minimum limit.

In order to satisfy the above three conditions, each vaccine allocation model should have an individual objective function, and the model needs to fulfill the objective/s. The proposed method allocates vaccines to reduce day-wise infection of COVID-19 and measures the accuracy with respect to the request and allocated vaccine. Hence, the models are developed according to this objective function. In this regard, we propose two models, proportional-based allocation and infection-based allocation, for day-wise vaccine allocation for each SUT. Both of the models fail to satisfy the last two conditions mentioned above. In order to satisfy all the conditions, we develop the hybrid model by combining the proportion and infection-based allocation models, where a fractional part of vaccine distribution is done by one model, and the remaining fractional part of vaccine distribution is done by another model, which is defined in Eq. (10). The two parameters PQ_i and IQ_i are the proportion and infection-based allocation vaccines of the SU_i respectively. The final allocated vaccine of the combined model is denoted by FQ_i which is estimated using Eq. (10).

$$FQ_i = \alpha * PQ_i + (1-\alpha) * IQ_i \tag{10}$$

Here $\alpha, (0 \le \alpha \le 1)$ be the cofactor that is used for the necessary adjustment of the final vaccine allocation.

4. Result and Discussion

The proposed vaccine distribution methods are applied for large-scale data sets and a wide range of data where the lowest limit is very low, whereas the maximum limit is very high. It is difficult to follow all the conditions and maintain all the rules during vaccine allocation due to the various impacts of different base, active, and relative factors. The proposed vaccine distribution methods try to manage various factors that impact vaccine distribution. However, none of the proposed methods are accurate, and the allocation of the vaccines among the SUTs is an ideal distribution. Then, the performance and efficiency of the methods are measured by introducing parameters like accuracy rate and various distribution grades. The accuracy rate (AR) is the ratio between the request of vaccine and allocation of vaccine, whereas the distribution grades are denoted as DG_{AC}, DG_{IR}, and DGTP, which indicates the vaccine distribution ratio with respect to active case, infection rate, and total population, respectively among the SUTs. The performance measuring parameters are formulated below.

$$AR_i = \frac{request\ vaccine_i}{Allocated\ vaccine_i} \tag{11}$$

$$DR_{AC}^{i} = \frac{Active\ Case_i}{Allocated\ vaccine_i} \tag{12}$$

$$DR_{TP}^{i} = \frac{Total\ Population_i}{Allocated\ vaccine_i} \tag{13}$$

$$DR_{IR}^{i} = \frac{Infection\ Rate_i}{Allocated\ vaccine_i} \tag{14}$$

The accuracy of the vaccine allocation among the SUTs is represented by the variable AR_i, $i = 1, 2, ..., n$. In the ideal situation, the magnitude of AR_i will be equal to 1, i.e., the allocated vaccine is equal to the requested amount, otherwise under allocation and over-allocation will occur according to conditions $AR_i \prec 1$ and $AR_i \succ 1$ respectively. The overall performance of the vaccine distribution is measured by the values of $AR_i, i \in n$ which belongs within the range $(1 \pm \beta)$, where the symbol β indicate tolerance of the vaccine allocation and the lowest tolerance value (β) of the vaccine allocation will be most acceptable. Similarly, the distribution grades DG_{AC}, DG_{IR} and DG_{TP} with respect to the active case, infection rate per million population, and total population are respectively the ratio of the active case and allocated vaccine, infection rate and allocated vaccine, total population and allocated vaccine, which are represented by the Eq. (12), Eq. (13) and Eq. (14) respectively. If those three types of grades lie within the narrow range, then the vaccine distribution is more acceptable.

The proportional-based distribution method is applied to distribute the ten core doses of the vaccine among thirty-six SUTs. Then, the SUT-wise request of vaccine and allocated vaccine by three vaccine distribution methods are represented using a bar plot in Figure 3. We realize that the vaccine distribution among the SUTs should be ideal when the demand for vaccines is moderate. However, the distribution of vaccines is not justified for the lowest as well as highest demand of the SUTs. The SUTs with indexes 5, 13, 29, 31, 33, 34, and 35 are allocated a quantity of vaccine, which is much more than the request of vaccine, whereas SUTs with indexes 12, 18, and 20 are allotted a limited quantity of vaccine in respect of request. The allocated vaccines for the SUTs with index 0, 4, 7, 8, 10, 15, 16, 17, 19, 22, 23, 24, 26, 32 are ideal in respect of 10% tolerance of request of vaccine. As per the performance report in Table 4, the accuracy rate range is very wide. Subsequently, the other three distribution grades, which are used to measure the performance of the vaccine allocation, are better than the accuracy rate, but displacement ranges are not narrow.

In the infection-based distribution method, the same quantity of vaccines is distributed among the thirty-six SUTs. This allocation method is more optimistic than proportional-based vaccine distribution. The infection-based vaccine distribution method is balanced over the demand, but a few situations fluctuated unexpectedly, which is shown in Figure 3.

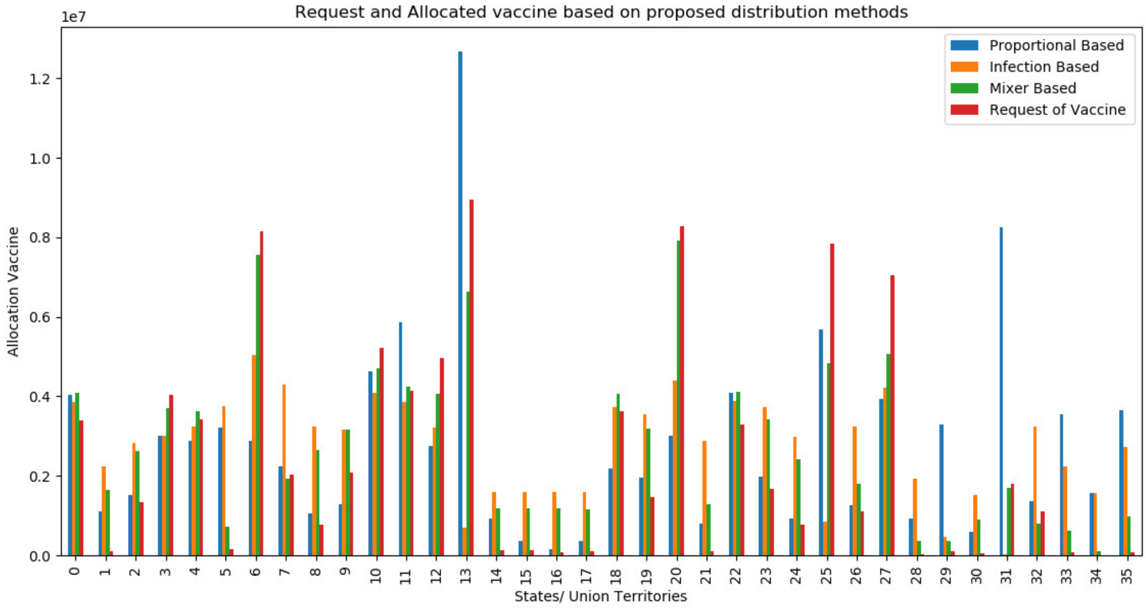

Fig. 3. Request for vaccines and distribution of vaccines by the three procedures among thirty-six SUTs.

As per the performance report computed in Table 4, the accuracy rate interval for the infection-based distribution method is [0.732, 218.12], which is much better than that of the proportional-based distribution. Also, it is observed that the interval of three distribution grades (DGAC, DGIR, DGTP) are almost similar for the proportional-based distribution method. According to the performance measure shown in Figure 3, both the proportional-based distribution and infection-based distribution methods are not found to be accurate since, in some of the SUTs, the allocated vaccines are more than the requested vaccines, and sometimes the allocated vaccines are less than the requested vaccines which are not desirable. However, the distribution of vaccines among the SUTs needs to be ideal, i.e., the distributed quantity of vaccines in SUTs should be nearly equal to the requested amount of vaccines. For this purpose, we have introduced the concept of a hybrid model to combine both of the proposed methods by partial fraction technique as given in Eq. (12). The hybrid approach is able to gather the advantages of both of the methods and try to balance the distribution among the SUTs. The hybrid-based vaccine allocation can improve the accuracy of SUTs such as the SUTs with indexes 6, 11, 13, and 31, where the proportional and infection-based vaccine allocation produces either over or under-allocated results. The vaccine distributions among the thirty-six SUTs as per the hybrid model are shown in Table 3, where the cofactor (α) value is between 0 and 1. The hybrid model-based distribution becomes a proportional based distribution when the cofactor (α) =1, similarly hybrid model-based vaccine distribution becomes infection base distribution when cofactor (α) =0. The desirable distribution of the hybrid model is considered when cofactor (α) =0.3. At that point, the accuracy rate of the hybrid model-based vaccine distribution is between 1.33 and 0.35, and the performance is much better than the other two methods proposed. The vaccine distributions of the hybrid model among the SUTs are shown in Figure 3. Although the distributed amount of vaccines among the SUTs is not proper or ideal, it is proportional to requested vaccines and may be accepted. In this hybrid vaccine allocation method, the allocated vaccine among sixteen SUTs is ideal. The vaccine distribution of the remaining SUTs is within 7% tolerance of the requested vaccine. The hybrid vaccine allocation among the SUTs based on cofactor (α) are shown in Table 3.

Table 3
Vaccine allocation among the SUTs for different value of α

		cofactor (α)										
States/UTs		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	Andhra Pradesh	4259009	4219503	4179996	4140490	4100983	4061477	3596070	3982464	3942958	3903451	3863945
2	Arunachal Pradesh	1238592	1338840	1439089	1539337	1639586	1739834	1716223	1940331	2040579	2140828	2241076
3	Assam	2504175	2536411	2568646	2600882	2633117	2665353	2447171	2729824	2762059	2794295	2826531
4	Bihar	4188622	4069964	3951307	3832649	3713991	3595334	3057814	3358018	3239360	3120703	3002045
5	Chhattisgarh	3868201	3805722	3743242	3680763	3618284	3555804	3106505	3430846	3368367	3305887	3243408
6	Goa	2036974	2209237	2381500	2553762	2726025	2898288	2866853	3242814	3415076	3587339	3759602
7	Gujarat	5890027	5805379	5720732	5636084	5551436	5466788	4793138	5297493	5212845	5128197	5043550
8	Haryana	3688686	3748869	3809052	3869235	3929417	3989600	3680914	4109966	4170149	4230332	4290514
9	Himachal Pradesh	2266383	2363494	2460605	2557716	2654827	2751938	2622411	2946160	3043271	3140383	3237494
10	Jharkhand	3150527	3151792	3153056	3154321	3155585	3156850	2843061	3159379	3160643	3161908	3163172
11	Karnataka	5115647	5013442	4911237	4809032	4706827	4604622	3990852	4400212	4298007	4195802	4093597
12	Kerala	4503298	4439318	4375338	4311359	4247379	4183399	3669089	4055439	3991459	3927480	3863500
13	Madhya Pradesh	4636309	4494715	4353121	4211526	4069932	3928338	3323113	3645150	3503555	3361961	3220367
14	Maharashtra	3930220	3606833	3283445	2960058	2636670	2313283	1596874	1666508	1343121	1019733	696346
15	Manipur	912627.8	980476	1048324	1116173	1184021	1251869	1228455	1387566	1455414	1523262	1591110
16	Meghalaya	916211	983476	1050741	1118006	1185271	1252536	1228180	1387066	1454331	1521596	1588862
17	Mizoram	883544	955737	1027930	1100123	1172316	1244509	1228347	1388895	1461088	1533281	1605474
18	Nagaland	886077	956417	1026758	1097098	1167439	1237779	1219512	1378460	1448800	1519141	1589481
19	Odisha	4273564	4218222	4162879	4107537	4052195	3996853	3514154	3886168	3830826	3775484	3720141
20	Punjab	2969587	3027333	3085079	3142825	3200571	3258317	3019105	3373810	3431556	3489302	3547048
21	Rajasthan	5621300	5497903	5374506	5251110	5127713	5004316	4318789	4757522	4634126	4510729	4387332
22	Sikkim	1541568	1675314	1809060	1942805	2076551	2210297	2189886	2477789	2611535	2745280	2879026
23	Tamil Nadu	4258598	4221108	4183618	4146128	4108638	4071148	3607798	3996168	3958678	3921187	3883697
24	Telangana	3203831	3255121	3306410	3357700	3408990	3460280	3191186	3562859	3614149	3665439	3716728
25	Tripura	2061056	2152342	2243628	2334913	2426199	2517485	2402665	2700057	2791342	2882628	2973914
26	Uttar Pradesh	4150880	3820052	3489224	3158396	2827569	2496741	1750825	1835085	1504257	1173429	842601.6
27	Uttarakhand	2515545	2589022	2662500	2735977	2809455	2882932	2704855	3029887	3103364	3176841	3250319
28	West Bengal	5627464	5485536	5343608	5201680	5059752	4917824	4213149	4633968	4492040	4350112	4208184
29	ANI[1]	997738	1089685	1181633	1273580	1365527	1457475	1449648	1641369	1733317	1825264	1917211
30	Chandigarh	314720	329602	344485	359367.3	374249.7	389132.1	372542.6	418897	433779.4	448661.8	463544.3
31	DNHDD[2]	812050	881693	951336	1020979	1090622	1160265	1148703	1299551	1369194	1438837	1508480
32	Delhi	818472	738391	658310	578228.4	498147.3	418066.1	256137.7	257903.7	177822.5	97741.35	17660.16
33	Jammu and Kashmir	2514606	2586974	2659341	2731709	2804076	2876444	2697351	3021179	3093547	3165915	3238282
34	Ladakh	1187116	1292063	1397009	1501956	1606903	1711850	1698085	1921743	2026690	2131636	2236583
35	Lakshadweep	795865	872113	948362	1024610	1100858	1177106	1173768	1329603	1405851	1482100	1558348
36	Puducherry	1460811	1587563	1714316	1841068	1967820	2094573	2075244	2348077	2474829	2601582	2728334

[1]Andaman and Nicobar Islands, [2]Dadra and Nagar Haveli and Daman and Diu.

Table 4

Performance report of the three vaccine distribution procedures.

	AR		DG_{TP}		DG_{IR}		DG_{AC}	
	max	min	max	min	max	min	max	min
Proportion Based Distribution	524.8	0.36	35.1	0.041	0.025	0.0006	0.04	0.00002
Infection Based Distribution	218.12	0.732	34.79	0.2	0.1001	0.00005	0.029	0.0007
Hybrid Distribution	1.33	0.35	24.5	0.018	0.021	0.00076	0.131	0.00009

5. Concluding Remarks

In this article, we have initially proposed the proportional and infection-based vaccine allocation methods among the states and union territories of India. India is a highly populated country, and the population density varies from a few persons to thousands of people in a Km^2. So, the infection rates and active cases vary among the various regions, whereas the medical services in some well-established locations are state-of-the-art. Then, the vaccine allocation process is very challenging, and it is difficult to maintain consistency and reduce the active cases, infection rate, and fatality rate. The proportional-based distribution method distributes the vaccine among the SUTs based on the present status of various parameters like the active case, infection rate, population density, total population, and request for the vaccine. The infection-based distribution method distributes the vaccine based on the rate of infection, population density, average contract hours per day, and vaccination ratio. As per our investigation report, it is observed that none of the distribution methods are able to distribute the vaccine exactly among the SUTs with respect to the request for the vaccine, total population, infection rate, and active cases. These two methods (proportional and infection-based vaccine allocation methods) distribute the vaccine in some SUTs quite accurately, whereas, in most of the SUTs, the vaccine distribution is either over-distributed or under-distributed. To improve the vaccine distribution performance among the SUTs, a hybrid method is proposed by combining the given two methods. The hybrid method provides a better result than the proposed two methods with respect to accuracy rate and various distribution grades. In the future, the vaccine distribution can be further improved by introducing machine learning concepts, where the relative importance of the individual parameters may be developed, and an intelligence model may be formulated to allocate the vaccine among the States and union territories.

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Conflicts of Interest
The authors declare no conflicts of interest.

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