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Maclaurin Symmetric Mean Aggregation Operators Based on Novel Frank T-Norm and T-Conorm for Picture Fuzzy Multiple-Attribute Group Decision-Making

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ABSTRACT

The proposed study introduces innovative Maclaurin symmetrical mean aggregation operators (MSMAO) in picture fuzzy multiple-attribute group decision-making. These operators are built upon a newly developed Frank t-norm (FTN) and t-conorm (TCN). Utilizing these novel operators aims to enhance the effectiveness of decision-making processes in scenarios characterized by picture-fuzzy information. The study explores these aggregation operators' potential applications and advantages within the context of multiple-attribute group decision-making under uncertainty. As a novel method to handle MAGDM difficulties, we build these aggregation operators (AOs) on PFs in this paper by utilizing the FTN and TCN under PFs information. In addition, PF values (PFVs) and their fundamental operations are created and shown. Picture fuzzy Frank weighted Maclaurin symmetrical mean (MFFWMSM) and Picture fuzzy Frank Maclaurin symmetrical mean (MFFMSM) operators are two AOs introduced and studied based on these operations. The induction approach theoretically and quantitatively verifies the newly created AOs' accuracy and dependability. The problem of project evaluation utilizing these proposed operators is thoroughly examined to provide further applications and investigate the sensitivity of these PS Frank (PFF) operators. The outcomes of employing these PFF operators are contrasted with a few AOs of PFVs that were previously in existence. The suggested strategy is dependable and effective based on comparison outcomes.

1. Introduction

Although it might be challenging to offer professional advice in ambiguous circumstances, Zadeh's fuzzy set (FS) [1] is a useful tool for articulating the membership degree (MD) of elements. Subsequently, the intuitionistic FS (IFS) framework was established by Atanassov [2], and the concept of FS was furnished with the non-membership degree (NG). The IFS framework was a flexible concept

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used extensively to study a range of real-world occurrences. Yager [3] strengthened the IFS framework by presenting the ideas of q-rung orthopair FS (qROFS) and Pythagorean FS (PyFS), which give considerable leeway in determining the MD and the NMD. In the IFS, PyFS, and qROFS frames, the MG and the NMG are the only two elements that characterize two aspects of human opinion. In addition to presenting the idea of a Picture Fuzzy Set (PFS), Cường [4] suggested adding two more elements, abstinence degree (AD) and NMD and NMD. According to them, the IFS concept does not adequately describe human perception as an opinion. As a result, information is lost when expressing human opinion using IFS, PyFS, and qROFS in the absence of AG and RG. Nevertheless, PFS is preferable to these three file systems because it recovers possibly lost information [5]. Applications such as parametric analysis and specific differential equations can gain from applying FS and its extension [6].

The MAGDM assists in choosing the best option from a list of options while considering several alternatives. The output of the MAGDM is cumulatively based on experts' opinions, which makes it more dependable than the multi-attribute decision-making (MADM). In contrast, the MADM's outcome is founded only on the opinions of one practiced. Consequently, the MAGDM is seen as an additional operative and well-organized decision-making method [7]. A multitude of industries, including business [8], have employed the MAGDM, dynamism [9], medicinal skill [10-12], and others. Numerous useful techniques for aggregating the information have been devised depending on the AOs. For instance, some significant AOs were introduced in [13], given the concept of the TOPSIS method for probabilistic linguistics on MAGDM. Khan *et al*., [14] developed the theoretical justifications for the empirically successful of MADM. Zhao *et al.,* [15] improved the TODIM method for IF MAGDM based on cumulative prospect theory. These techniques thoroughly aggregate the knowledge of alternatives, unlike conventional AOs. Due to their broad applicability, the AOs have recently emerged as the most intriguing research area in MADM and MAGDM.

Using MADM, Wei *et al*., [16] create the Pythagorean fuzzy Maclaurin symmetric mean operator. Ullah *et al.,* [17] introduced the idea of fuzzy macular images in symmetric mean operators. [18] Data aggregators known as AOs were introduced for interval-valued IFS influence MSM procedures. The disadvantage of these AOs is that their original operating laws were conventional. Qin *et al*., [19] consider that MADM is built on PF Archimedean power MSM operators. Interval-valued PFMSM operator is developed by Ashraf *et al*., [20] and applied to MADM. A few new MSM operators for Q-Rung PF numbers and how they work with MAGAD are presented in [21]. Also, 2-tuple Linguistic MSM Operators and MAGDM Application are presented in [22]. The hesitant PF Schweizer-Sklar MSM operator was developed by Chen and Ye [23]. Mahmood and ur Rehman [24] in light of the bipolar complex fuzzy MSM operator on MADM technique. Ali and Mahmood [25] T-MSM operators and their uses with sophisticated Q-Rung orthopair fuzzy sets. Garg and Arora [26] Archimedean-based generalized MSM aggregation operators. Qin *et al.,* [27] in light of the theory behind hesitant fuzzy MSM operators and how it applies to MADM. Hussain *et al.,* [28] MSM aggregation operators founded on the innovative Frank T-norm and T-conorm. Based on Schweizer-Sklar operators, Wang and Liu [29] developed a few MSM aggregation operators. Q-Rung orthopair fuzzy MSM operator by Wei *et al.,* [30]. Azeem and Nadeem [31] considered the ideas behind the metric-based resolvability of aromatic polycyclic hydrocarbons. Azeem *et al.,* [32,33] created M-Polynomials utilizing nanostructures and presented localization of hexagonal cellular networks with generalization.

The ability of PFS to characterize unclear information utilizing NMD, AD, and MD indicates the importance of PFS in tackling the problems of MADM, pattern recognition, clustering, and medical diagnosis. Furthermore, because they merely offer information aggregation and do not correlate the supplied data, it has been noted that traditional aggregation operators have partially lost their trust. As a framework for handling the ambiguity and uncertainty in the evidence, PFS has grown in favor among academics. Frank's domestic operating rules are dependable and adaptable, making them a good choice [34-36] often employed by researchers in MAGDM, as seen in [37]. By correlating the input data, MSM operators get around this drawback of conventional aggregation operators. In this paper, we aim to develop the notion of MSM operators for PFSs, considering the diverse structure of PFSs where four aspects of uncertain information can be described that minimize the likelihood of information loss and the significance of MSM operators. By simultaneously taking into account the relationship between more than two input arguments, the recently developed PFMSM operators can aggregate uncertain information. This kind of AO minimizes information loss while giving us flexibility and robustness. The following is a summary of this paper's primary contributions:

i. This paper aims to discuss certain related features and suggest AOs for PF information based on Frank TN and TCN, or PFFMMS and PFFWMSM.
ii. Using the PFFMSM and PFFWMSM operators as a foundation, we created the Picture Fuzzy MAGDM approach.
iii. By resolving investment choice problems, we evaluated the suitability of our suggested MAGDM approach based on aggregation functions.
iv. The suggested approach is carried out through parameter analysis and comparison study with current approaches to demonstrate its efficacy and dependability.

There are seven sections in this article. Basic terminology useful in understanding the material is introduced in Section 2. The Frank TN and TCN sum and product of two PFS were defined in Section 3. We also provide the Frank operation properties on PFVs. In Section 4, we explore the fundamental features of PFFMSM operators and develop them. PFFWMSM operators are introduced in Section 5. The methodology for using PFFWMSM operators in the MAGDM issue is presented in Section 6. We use the PFFWMSM operator on a set of PFVs as an example. We examine the behaviors of the PFFWMSM operator for various parameter values and conduct a comparative analysis with other AOs. In Section 7, we draw our ultimate conclusions.

2. Methodology

2.1 We can understand the basic concepts of PFS, MSM, FTN, and TCM.

Definition 1 [37]. Let the $\ddot{\text{ך}}$ is an all-inclusive set. Then a PVFS $\dot{U}$ characterized are

$$\dot{\mathrm{U}} = \left\{\langle \mathrm{P}, \mathcal{r}_{\dot{\mathcal{B}}}(\mathrm{P}), \mathcal{a}_{\dot{\mathcal{B}}}(\mathrm{P}), \mathcal{b}_{\dot{\mathcal{B}}}(\mathrm{P})\rangle | \mathrm{P} \in \ddot{\text{ך}}\right\}$$

Where $\mathcal{r}_{\dot{\mathcal{B}}}: \ddot{\text{ך}} \to [0,1], \mathcal{a}_{\dot{\mathcal{B}}}: \ddot{\text{ך}} \to [0,1], \mathcal{b}_{\dot{\mathcal{B}}}: \ddot{\text{ך}} \to [0,1]$ are DM and DN, individually, with $\mathcal{r}_{\dot{\mathcal{B}}}+\mathcal{a}_{\dot{\mathcal{B}}}+\mathcal{b}_{\dot{\mathcal{B}}} \in [0,1]$. The reluctance was not entirely settled by $1 - \mathcal{r}_{\dot{\mathcal{B}}} - \mathcal{a}_{\dot{\mathcal{B}}} - \mathcal{b}_{\dot{\mathcal{B}}}$.

Definition 2 [37]. Assume $\acute{\mathrm{R}} = (\mathcal{r}_{\dot{\mathcal{B}}}, \mathcal{a}_{\dot{\mathcal{B}}}, \mathcal{b}_{\dot{\mathcal{B}}})$ be a PFV. The score capability of is characterized as follows,

$\mathrm{Sc}(\acute{\mathrm{R}}) = \mathcal{r}_{\dot{\mathcal{B}}} - \mathcal{a}_{\dot{\mathcal{B}}} - \mathcal{b}_{\dot{\mathcal{B}}}$ (1)

$\mathrm{Sc}(\acute{\mathrm{R}}) \in [-1,1]$. Smaller the $\mathrm{Sc}(\acute{\mathrm{R}})$ is the more modest the PFV $\acute{\mathrm{R}}$

It was found that some PFVs couldn't be situated with the help of the scoring capacity described previously [38]—the possibility of the grade which can be depicted below.

Definition 3 [37]. Assume $\acute{\mathrm{R}} = (\mathcal{r}_{\dot{\mathcal{B}}}, \mathcal{a}_{\dot{\mathcal{B}}}, \mathcal{b}_{\dot{\mathcal{B}}})$ be a PFV. The AD of $\acute{\mathrm{R}}$ is characterized as the accompanying $\mathrm{AD}(\mathrm{E}) \in [0,1]$

$\mathrm{AD}(\acute{\mathrm{R}}) = \mathcal{r}_{\dot{\mathcal{B}}} + \mathcal{a}_{\dot{\mathcal{B}}} + \mathcal{b}_{\dot{\mathcal{B}}}$ (2)

$\mathrm{AD}(\acute{\mathrm{R}}) \in [0,1]$.

Definition 4 [34]. The Frank mapping are TN and TCN, such as form $[0,1]^2$to $[0,1]$

$$\mathrm{T_F}(\theta,\psi)=\mathrm{Log}_\alpha\left(1+\frac{(\alpha^\theta-1)(\alpha^\psi-1)}{\alpha-1}\right),\theta,\psi\in[0,1],\alpha>1$$

$$\mathrm{S_F}(\theta,\psi)=1-\mathrm{Log}_\alpha\left(1+\frac{(\alpha^{1-\theta}-1)(\alpha^{1-\psi}-1)}{\alpha-1}\right),\theta,\psi\in[0,1],\alpha>1.$$

$$\mathrm{P_F}(\theta,\psi)=1-\mathrm{Log}_\alpha\left(1+\frac{(\alpha^{1-\theta}-1)(\alpha^{1-\psi}-1)}{\alpha-1}\right),\theta,\psi\in[0,1],\alpha>1$$

Definition 5 [34]. Let $\acute{R}_1=(r_{\dot{B}1},a_{\dot{B}1},b_{\dot{B}1})$, $\acute{R}_2=(r_{\dot{B}2},a_{\dot{B}2},b_{\dot{B}2})$ and $\acute{R}=(r_{\dot{B}},a_{\mathcal{B}},b_{\mathcal{B}})$ are PFVs and $n>0$, PFVs can be based on the Frank TN and TCN,

$$\acute{R}_1\oplus_F\acute{R}_2=\left(\begin{array}{c}1-\mathrm{Log}_\alpha\left(1+\frac{\left(\alpha^{1-r_{\dot{B}1}}-1\right)\left(\alpha^{1-r_{\dot{B}2}}-1\right)}{\alpha-1}\right),\\ \mathrm{Log}_\alpha\left(1+\frac{(\alpha^{a_{\dot{B}1}}-1)(\alpha^{a_{\dot{B}2}}-1)}{\alpha-1}\right),\mathrm{Log}_\alpha\left(1+\frac{\left(\alpha^{b_{\dot{B}1}}-1\right)\left(\alpha^{b_{\dot{B}2}}-1\right)}{\alpha-1}\right)\end{array}\right) \quad (3)$$

$$\acute{R}_1\otimes_F\acute{R}_2=\left(\begin{array}{c}\mathrm{Log}_\alpha\left(1+\frac{(\alpha^{r_{\dot{B}1}}-1)(\alpha^{r_{\dot{B}2}}-1)}{\alpha-1}\right),1-\mathrm{Log}_\alpha\left(1+\frac{\left(\alpha^{1-a_{\dot{B}1}}-1\right)\left(\alpha^{1-a_{\dot{B}1}}-1\right)}{\alpha-1}\right),\\ 1-\mathrm{Log}_\alpha\left(1+\frac{\left(\alpha^{1-b_{\dot{B}1}}-1\right)\left(\alpha^{1-b_{\dot{B}1}}-1\right)}{\alpha-1}\right)\end{array}\right) \quad (4)$$

$$n.\acute{R}=\left(1-\mathrm{Log}_\alpha\left(1+\frac{(\alpha^{1-r}-1)^n}{(\alpha-1)^{n-1}}\right),\mathrm{Log}_\alpha\left(1+\frac{(\alpha^{a}-1)^n}{(\alpha-1)^{n-1}}\right),\mathrm{Log}_\alpha\left(1+\frac{(\alpha^{b}-1)^n}{(\alpha-1)^{n-1}}\right)\right) \quad (5)$$

$$\acute{R}^n=\left(\begin{array}{c}\mathrm{Log}_\alpha\left(1+\frac{(\alpha^{r}-1)^n}{(\alpha-1)^{n-1}}\right)\\ ,1-\mathrm{Log}_\alpha\left(1+\frac{(\alpha^{1-a}-1)^n}{(\alpha-1)^{n-1}}\right),1-\mathrm{Log}_\alpha\left(1+\frac{(\alpha^{1-b}-1)^n}{(\alpha-1)^{n-1}}\right)\end{array}\right) \quad (6)$$

Definition 6 [39]. Let $\mathrm{h_m}(\mathrm{m}=1,2,3,\cdots,n)$ the number positive, Than MSM operator is MSM are.

$$\mathrm{MSM}(\mathrm{h}_1,\mathrm{h}_2,\mathrm{h}_3,\cdots\mathrm{h}_n)=\left(\frac{\sum_{1\le \mathrm{m}_1<\mathrm{m}_2<\mathrm{m}_3<\cdots<\mathrm{m}_\varrho\le n}\prod_{\mathrm{l}=1}^{\varrho}\mathrm{h}_{\mathrm{m_l}}}{\mathcal{C}_\varrho^n}\right)^{1/\varrho} \quad (7)$$

Where, $\mathcal{C}_\varrho^n$ is the binomial coefficient, $1\le\mathrm{m}_1<\mathrm{m}_2<\mathrm{m}_3<\cdots<\mathrm{m}_\varrho\le n$ represents the $\varrho-$tuple real number of positive.

3. The PFVs Symmetric Frank Maclaurin Mean Operators

The segment comprises the advancement of the PFFMSM operator and depends on the Frank operator on PFVs. This segment can also concentrate on a few positive properties of PFMSM.

Definition 7. Let $\acute{R}_\iota$ $(\iota=1,2,3\cdots,n)$ are assortment of the PFVs, Then PFFMSM: $\ddot{\daleth}^n\to\ddot{\daleth}$ operator is characterized,

$$\mathrm{PFFMSM}(\acute{R}_1,\acute{R}_2,\acute{R}_3,\cdots\acute{R}_n)=\left(\frac{\oplus_{F\,1\le\iota_1<\iota_2<\iota_3<\cdots<\iota_\varrho\le n}\otimes_{F\,j=1}^{\varrho}\alpha_{\iota_j}\acute{R}_{\iota_j}}{\mathcal{C}_\varrho^{\mathrm{n}}}\right)^{1/\varrho} \quad (8)$$

Where, $\mathcal{C}_\varrho^n$ is the binomial coefficient, $1\le\iota_1<\iota_2<\iota_3<\cdots<\iota_\varrho\le\mathrm{n}$ represents the $\varrho-$tuple PFVs.

The Frank operations can be characterized above; we Can demonstrate a few crucial properties of PFFMSM operators. Theorem 1 can be aggregated again in PFV.

Theorem 1. Let $\acute{\mathrm{R}}_\iota = (r_\iota, a_\iota, b_\iota)$ $(\iota = 1,2,3,\cdots,n)$ the assortment of PFVs and Then the PFV can be aggregated by the PFFMSM operator and $\mathrm{FFMSM}(\acute{\mathrm{R}}_1, \acute{\mathrm{R}}_2, \acute{\mathrm{R}}_3, \cdots \acute{\mathrm{R}}_n) =$

$$\left(\begin{array}{c}
\mathrm{Log}_\alpha\left(1+\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\prod_{1\le\imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\left(\alpha^{1-\left(\mathrm{Log}_\alpha\left(1+\frac{\prod_{\imath=1}^{n}(\alpha^{r_\imath}-1)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)}-1\right)^{\frac{1}{C_\varrho^n}}(\alpha-1)^{1-\frac{1}{C_\varrho^n}}\right)}-1\right)^{\frac{1}{\varrho}}(\alpha-1)^{1-\frac{1}{\varrho}}\right), \\
1-\mathrm{Log}_\alpha\left(1+\left(\alpha^{\mathrm{Log}_\alpha\left(1+\left(\alpha^{\mathrm{Log}_\alpha\left(1+\prod_{1\le\imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\left(\alpha^{\left(1-\mathrm{Log}_\alpha\left(1+\frac{\prod_{\imath=1}^{n}(\alpha^{1-a_\imath}-1)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)}-1\right)^{\frac{1}{C_\varrho^n}}(\alpha-1)^{1-\frac{1}{C_\varrho^n}}\right)}-1\right)^{\frac{1}{C_\varrho^n}}(\alpha-1)^{1-\frac{1}{C_\varrho^n}}\right), \\
1-\mathrm{Log}_\alpha\left(1+\left(\alpha^{\mathrm{Log}_\alpha\left(1+\left(\alpha^{\mathrm{Log}_\alpha\left(1+\prod_{1\le\imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\left(\alpha^{\left(1-\mathrm{Log}_\alpha\left(1+\frac{\prod_{\imath=1}^{n}(\alpha^{1-b_\imath}-1)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)}-1\right)^{\frac{1}{C_\varrho^n}}(\alpha-1)^{1-\frac{1}{C_\varrho^n}}\right)}-1\right)^{\frac{1}{C_\varrho^n}}(\alpha-1)^{1-\frac{1}{C_\varrho^n}}\right)
\end{array}\right)$$

Proof: This theorem, we can be prove $\bigotimes_{\mathrm{F}\,j=1}^{\varrho} \acute{\mathrm{R}}_{\imath_j}$ as follows

$$\bigotimes_{\mathrm{F}\,j=1}^{\varrho} \acute{\mathrm{R}}_{\imath_j} = \left(\begin{array}{c}
\mathrm{Log}_\alpha\left(1+\left(\prod_{\imath=1}^{n}(\alpha^{r_\imath}-1)\right)(\alpha-1)^{1-n}\right), \\
1-\mathrm{Log}_\alpha\left(1+\left(\prod_{\imath=1}^{n}(\alpha^{1-a_\imath}-1)\right)(\alpha-1)^{1-n}\right), \\
1-\mathrm{Log}_\alpha\left(1+\left(\prod_{\imath=1}^{n}(\alpha^{1-b_\imath}-1)\right)(\alpha-1)^{1-n}\right)
\end{array}\right)$$

This is a PFV. Now, we calculate $\bigoplus_{\mathrm{F}\,1\le\imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n} \bigotimes_{\mathrm{F}\,j=1}^{\varrho} \acute{\mathrm{R}}_{\imath_j}$ as follows

$$\bigoplus_{\mathrm{F}\,1\le \imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\ \bigotimes_{\mathrm{F}\,j=1}^{\varrho}\acute{\mathrm{R}}_{\imath_j} = \left(\begin{array}{l} 1-\mathrm{Log}_\alpha\left(1+\prod_{1\le \imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\left(\alpha^{1-\left(\mathrm{Log}_\alpha\left(1+\frac{\prod_{\imath=1}^{n}\left(\alpha^{r_\imath}-1\right)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right), \\ \mathrm{Log}_\alpha\left(1+\prod_{1\le \imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\left(\alpha^{\left(1-\mathrm{Log}_\alpha\left(1+\frac{\prod_{\imath=1}^{n}\left(\alpha^{1-a_\imath}-1\right)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right), \\ \mathrm{Log}_\alpha\left(1+\prod_{1\le \imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\left(\alpha^{\left(1-\mathrm{Log}_\alpha\left(1+\frac{\prod_{\imath=1}^{n}\left(\alpha^{1-b_\imath}-1\right)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right) \end{array}\right)$$

$\bigoplus_{\mathrm{F}\,1\le \imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\ \bigotimes_{\mathrm{F}\,j=1}^{\varrho}\acute{\mathrm{R}}_{\imath_j}$ is also clearly a PFV. Now, we calculate $\left(\frac{\bigoplus_{\mathrm{F}\,1\le \imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\ \bigotimes_{\mathrm{F}\,j=1}^{\varrho}\acute{\mathrm{R}}_{\imath_j}}{\mathcal{C}_\varrho^n}\right)^{1/\varrho}$ as follows

$$\left(\frac{\bigoplus_{\mathrm{F}\,1\le \imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\ \bigotimes_{\mathrm{F}\,j=1}^{\varrho}\acute{\mathrm{R}}_{\imath_j}}{\mathcal{C}_\varrho^n}\right)^{1/\varrho} = \left(\begin{array}{l} \mathrm{Log}_\alpha\left(1+\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\prod_{1\le \imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\left(\alpha^{1-\left(\mathrm{Log}_\alpha\left(1+\frac{\prod_{\imath=1}^{n}(\alpha^{r_\imath}-1)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)}-1\right)^{\frac{1}{\mathcal{C}_\varrho^n}}(\alpha-1)^{1-\frac{1}{\mathcal{C}_\varrho^n}}\right)}-1\right)^{\frac{1}{\varrho}}(\alpha-1)^{1-\frac{1}{\varrho}}\right), \\ 1-\mathrm{Log}_\alpha\left(1+\left(\alpha^{\mathrm{Log}_\alpha\left(1+\left(\alpha^{\mathrm{Log}_\alpha\left(1+\prod_{1\le \imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\left(\alpha^{\left(1-\mathrm{Log}_\alpha\left(1+\frac{\prod_{\imath=1}^{n}(\alpha^{1-a_\imath}-1)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)}-1\right)^{\frac{1}{\mathcal{C}_\varrho^n}}(\alpha-1)^{1-\frac{1}{\mathcal{C}_\varrho^n}}\right)}-1\right)^{\frac{1}{\varrho}}(\alpha-1)^{1-\frac{1}{\varrho}}\right), \\ 1-\mathrm{Log}_\alpha\left(1+\left(\alpha^{\mathrm{Log}_\alpha\left(1+\left(\alpha^{\mathrm{Log}_\alpha\left(1+\prod_{1\le \imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\left(\alpha^{\left(1-\mathrm{Log}_\alpha\left(1+\frac{\prod_{\imath=1}^{n}(\alpha^{1-b_\imath}-1)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)}-1\right)^{\frac{1}{\mathcal{C}_\varrho^n}}(\alpha-1)^{1-\frac{1}{\mathcal{C}_\varrho^n}}\right)}-1\right)^{\frac{1}{\varrho}}(\alpha-1)^{1-\frac{1}{\varrho}}\right) \end{array}\right)$$

So the proof can be completed.

Theorem 2 (Idempotency). Let $\acute{\mathrm{R}}_\imath = (r_\imath, a_\imath, b_\imath)$ be the three of PFVs, and if $\acute{\mathrm{R}}_\imath = (r_\imath, a_\imath, b_\imath) = \acute{\mathrm{R}} = (r, a, b)\ \forall\ (\imath = 1,2,3,\cdots,n)$. Then $\mathrm{PFFMSM}\left(\acute{\mathrm{R}}_1, \acute{\mathrm{R}}_2, \acute{\mathrm{R}}_3, \cdots \acute{\mathrm{R}}_n\right) = \acute{\mathrm{R}}$ is called the idempotent.

Proof: As $\acute{\mathrm{R}}_\imath = (r_\imath, a_\imath, b_\imath) = (r, a, b) = \acute{\mathrm{R}}\ \forall\ (\imath = 1,2,3,\cdots,n)$, by the Theorem 1, we have

$$
\begin{aligned}
&\mathrm{PFFMSM}\left(\acute{\mathrm{R}}_1, \acute{\mathrm{R}}_2, \acute{\mathrm{R}}_3, \cdots \acute{\mathrm{R}}_n\right) \\
&= \left(\begin{array}{l}
\mathrm{Log}_\alpha\left(1+\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\prod_{1\le\imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\left(\alpha^{1-\left(\mathrm{Log}_\alpha\left(1+\frac{\prod_{\imath=1}^{n}\left(\alpha^{r}-1\right)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)}-1\right)^{\frac{1}{\mathcal{C}_\varrho^n}}(\alpha-1)^{1-\frac{1}{\mathcal{C}_\varrho^n}}\right)}-1\right)^{\frac{1}{\varrho}}(\alpha-1)^{1-\frac{1}{\varrho}}\right), \\
1-\mathrm{Log}_\alpha\left(1+\left(\alpha^{\mathrm{Log}_\alpha\left(1+\left(\alpha^{\mathrm{Log}_\alpha\left(1+\prod_{1\le\imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\left(\alpha^{\left(1-\mathrm{Log}_\alpha\left(1+\frac{\prod_{\imath=1}^{n}\left(\alpha^{1-a}-1\right)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)}-1\right)^{\frac{1}{\mathcal{C}_\varrho^n}}(\alpha-1)^{1-\frac{1}{\mathcal{C}_\varrho^n}}\right)}-1\right)^{\frac{1}{\varrho}}(\alpha-1)^{1-\frac{1}{\varrho}}\right), \\
1-\mathrm{Log}_\alpha\left(1+\left(\alpha^{\mathrm{Log}_\alpha\left(1+\left(\alpha^{\mathrm{Log}_\alpha\left(1+\prod_{1\le\imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\left(\alpha^{\left(1-\mathrm{Log}_\alpha\left(1+\frac{\prod_{\imath=1}^{n}\left(\alpha^{1-b}-1\right)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)}-1\right)^{\frac{1}{\mathcal{C}_\varrho^n}}(\alpha-1)^{1-\frac{1}{\mathcal{C}_\varrho^n}}\right)}-1\right)^{\frac{1}{\varrho}}(\alpha-1)^{1-\frac{1}{\varrho}}\right)
\end{array} \right)
\end{aligned}
$$

$$= \left(\begin{array}{c} \mathrm{Log}_{\alpha}\left(1+\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\prod_{1\le \iota_1<\iota_2<\iota_3<\cdots<\iota_{\varrho}\le n}\left(\alpha^{1-\left(\mathrm{Log}_{\alpha}\left(1+\frac{(\alpha^{r}-1)^{n}}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)}-1\right)^{\frac{1}{C_{\varrho}^{n}}}(\alpha-1)^{1-\frac{1}{C_{\varrho}^{n}}}\right)}-1\right)^{\frac{1}{\varrho}}(\alpha-1)^{1-\frac{1}{\varrho}}\right), \\ 1-\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{\mathrm{Log}_{\alpha}\left(1+\prod_{1\le \iota_1<\iota_2<\iota_3<\cdots<\iota_{\varrho}\le n}\left(\alpha^{\left(1-\mathrm{Log}_{\alpha}\left(1+\frac{(\alpha^{1-a}-1)^{n}}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)}-1\right)^{\frac{1}{C_{\varrho}^{n}}}(\alpha-1)^{1-\frac{1}{C_{\varrho}^{n}}}\right)}-1\right)^{\frac{1}{\varrho}}(\alpha-1)^{1-\frac{1}{\varrho}}\right), \\ 1-\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{\mathrm{Log}_{\alpha}\left(1+\prod_{1\le \iota_1<\iota_2<\iota_3<\cdots<\iota_{\varrho}\le n}\left(\alpha^{\left(1-\mathrm{Log}_{\alpha}\left(1+\frac{(\alpha^{1-b}-1)^{n}}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)}-1\right)^{\frac{1}{C_{\varrho}^{n}}}(\alpha-1)^{1-\frac{1}{C_{\varrho}^{n}}}\right)}-1\right)^{\frac{1}{\varrho}}(\alpha-1)^{1-\frac{1}{\varrho}}\right) \end{array} \right)$$

$$= \left(\begin{array}{c} \mathrm{Log}_{\alpha}\left(1+(\alpha^{r}-1)^{\frac{1}{\varrho}}(\alpha-1)^{1-\frac{1}{\varrho}}\right), \\ 1-\mathrm{Log}_{\alpha}\left(1+(\alpha^{1-a}-1)^{\frac{1}{\varrho}}(\alpha-1)^{1-\frac{1}{\varrho}}\right), 1-\mathrm{Log}_{\alpha}\left(1+(\alpha^{1-b}-1)^{\frac{1}{\varrho}}(\alpha-1)^{1-\frac{1}{\varrho}}\right) \end{array} \right)$$

$$= (r, a, b) = \acute{R}$$

So the proof can be completed.

Theorem 3 (Monotonicity). The $\acute{R}_{\iota} = (r_{\iota}, a_{\iota}, b_{\iota})$ and $\acute{R}_{\iota}^{a} = (r_{\iota}^{a}, a_{\iota}^{a}, b_{\iota}^{a})$ be the three of PFVs and if $\acute{R}_{\iota} \ge \acute{R}_{\iota}^{a}$. Then $\mathrm{PFFMSM}(\acute{R}_1, \acute{R}_2, \acute{R}_3, \cdots \acute{R}_n) \ge \mathrm{PFFMSM}(\acute{R}_1^{a}, \acute{R}_2^{a}, \acute{R}_3^{a}, \cdots \acute{R}_n^{a})$.

Proof: As stated that $\acute{R}_{\iota} \ge \acute{R}_{\iota}^{a}$ so, for PFVs $r_{\iota} \ge r_{\iota}^{a}$, $b_{\iota} \le b_{\iota}^{a}$ and $b_{\iota} \le b_{\iota}^{a}$. We can write

$$\mathrm{Log}_{\alpha}\left(1+\left(\prod_{\iota=1}^{n}(\alpha^{r_{\iota}}-1)\right)(\alpha-1)^{1-n}\right) \ge \mathrm{Log}_{\alpha}\left(1+\left(\prod_{\iota=1}^{n}\left(\alpha^{r_{\iota}^{a}}-1\right)\right)(\alpha-1)^{1-n}\right)$$

And

$$1-\mathrm{Log}_{\alpha}\left(1+\left(\prod_{\imath=1}^{n}(\alpha^{1-a_\imath}-1)\right)(\alpha-1)^{1-n}\right)\leq 1-\mathrm{Log}_{\alpha}\left(1+\left(\prod_{\imath=1}^{n}\left(\alpha^{1-a_\imath^{\mathrm{a}}}-1\right)\right)(\alpha-1)^{1-n}\right)$$

$$1-\mathrm{Log}_{\alpha}\left(1+\left(\prod_{\imath=1}^{n}\left(\alpha^{1-b_\imath}-1\right)\right)(\alpha-1)^{1-n}\right)\leq 1-\mathrm{Log}_{\alpha}\left(1+\left(\prod_{\imath=1}^{n}\left(\alpha^{1-b_\imath^{\mathrm{a}}}-1\right)\right)(\alpha-1)^{1-n}\right)$$

Further, we have

$$1-\mathrm{Log}_{\alpha}\left(1+\prod_{1\leq\imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\leq n}\left(\alpha^{1-\left(\mathrm{Log}_{\alpha}\left(1+\frac{\prod_{\imath=1}^{n}(\alpha^{r_\imath}-1)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)$$
$$\geq 1-\mathrm{Log}_{\alpha}\left(1+\prod_{1\leq\imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\leq n}\left(\alpha^{1-\left(\mathrm{Log}_{\alpha}\left(1+\frac{\prod_{\imath=1}^{n}\left(\alpha^{r_\imath^{\mathrm{a}}}-1\right)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)$$

And

$$\mathrm{Log}_{\alpha}\left(1+\prod_{1\leq\imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\leq n}\left(\alpha^{1-\left(\mathrm{Log}_{\alpha}\left(1+\frac{\prod_{\imath=1}^{n}(\alpha^{1-a_\imath}-1)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)$$
$$\leq \mathrm{Log}_{\alpha}\left(1+\prod_{1\leq\imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\leq n}\left(\alpha^{1-\left(\mathrm{Log}_{\alpha}\left(1+\frac{\prod_{\imath=1}^{n}\left(\alpha^{1-a_\imath^{\mathrm{a}}}-1\right)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)$$

$$\mathrm{Log}_{\alpha}\left(1+\prod_{1\leq\imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\leq n}\left(\alpha^{1-\left(\mathrm{Log}_{\alpha}\left(1+\frac{\prod_{\imath=1}^{n}(\alpha^{1-b_\imath}-1)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)$$
$$\leq \mathrm{Log}_{\alpha}\left(1+\prod_{1\leq\imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\leq n}\left(\alpha^{1-\left(\mathrm{Log}_{\alpha}\left(1+\frac{\prod_{\imath=1}^{n}\left(\alpha^{1-b_\imath^{\mathrm{a}}}-1\right)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)$$

Then we have

$$\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\alpha^{1-\left(\mathrm{Log}_{\alpha}\left(1+\frac{\prod_{i=1}^{n}\left(\alpha^{\varkappa_i}-1\right)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)}-1\right)^{\frac{1}{C_{\varrho}^{n}}}(\alpha-1)^{1-\frac{1}{C_{\varrho}^{n}}}\right)}-1\right)^{\frac{1}{\varrho}}(\alpha-1)^{1-\frac{1}{\varrho}}\right)$$

$$\geq \mathrm{Log}_{\alpha}\left(1+\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\alpha^{1-\left(\mathrm{Log}_{\alpha}\left(1+\frac{\prod_{i=1}^{n}\left(\alpha^{\varkappa_i^{a}}-1\right)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)}-1\right)^{\frac{1}{C_{\varrho}^{n}}}(\alpha-1)^{1-\frac{1}{C_{\varrho}^{n}}}\right)}-1\right)^{\frac{1}{\varrho}}(\alpha-1)^{1-\frac{1}{\varrho}}\right)$$

And

$$1-\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{\mathrm{Log}_{\alpha}\left(1+\prod_{1\le \imath_1<\imath_2<\imath_3<\cdots<\imath_{\varrho}\le n}\left(\alpha^{\left(1-\mathrm{Log}_{\alpha}\left(1+\frac{\prod_{\imath=1}^{n}\left(\alpha^{1-a_{\imath}}-1\right)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)}-1\right)^{\frac{1}{C_{\varrho}^{n}}}(\alpha-1)^{1-\frac{1}{C_{\varrho}^{n}}}\right)}-1\right)^{\frac{1}{\varrho}}(\alpha-1)^{1-\frac{1}{\varrho}}\right)$$

$$\leq 1-\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{\mathrm{Log}_{\alpha}\left(1+\prod_{1\le \imath_1<\imath_2<\imath_3<\cdots<\imath_{\varrho}\le n}\left(\alpha^{\left(1-\mathrm{Log}_{\alpha}\left(1+\frac{\prod_{\imath=1}^{n}\left(\alpha^{1-a_{\imath}^{a}}-1\right)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)}-1\right)^{\frac{1}{C_{\varrho}^{n}}}(\alpha-1)^{1-\frac{1}{C_{\varrho}^{n}}}\right)}-1\right)^{\frac{1}{\varrho}}(\alpha-1)^{1-\frac{1}{\varrho}}\right)$$

$$1-\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{\mathrm{Log}_{\alpha}\left(1+\prod_{1\le\imath_1<\imath_2<\imath_3<\cdots<\imath_{\varrho}\le n}\left(\alpha^{\left(1-\mathrm{Log}_{\alpha}\left(1+\frac{\prod_{\imath=1}^{n}\left(\alpha^{1-\mathscr{b}_{\imath}}-1\right)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)}-1\right)^{\frac{1}{\mathcal{C}_{\varrho}^{n}}}(\alpha-1)^{1-\frac{1}{\mathcal{C}_{\varrho}^{n}}}\right)}-1\right)^{\frac{1}{\varrho}}(\alpha-1)^{1-\frac{1}{\varrho}}\right)$$

$$\le 1-\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{\mathrm{Log}_{\alpha}\left(1+\prod_{1\le\imath_1<\imath_2<\imath_3<\cdots<\imath_{\varrho}\le n}\left(\alpha^{\left(1-\mathrm{Log}_{\alpha}\left(1+\frac{\prod_{\imath=1}^{n}\left(\alpha^{1-\mathscr{b}_{\imath}^{\mathrm{a}}}-1\right)}{(\alpha-1)^{\varrho-1}}\right)\right)}-1\right)(\alpha-1)^{1-\varrho}\right)}-1\right)^{\frac{1}{\mathcal{C}_{\varrho}^{n}}}(\alpha-1)^{1-\frac{1}{\mathcal{C}_{\varrho}^{n}}}\right)}-1\right)^{\frac{1}{\varrho}}(\alpha-1)^{1-\frac{1}{\varrho}}\right)$$

Theorem 4 (Boundedness). The $\acute{\mathrm{R}}_{\imath}=(\mathscr{r}_{\imath},\mathscr{a}_{\imath},\mathscr{b}_{\imath})$ be the three of PFVs, and if $\acute{\mathrm{R}}^{-}=(\min\mathscr{r}_{\imath},\max\mathscr{a}_{\imath},\max\mathscr{b}_{\imath})$, $\acute{\mathrm{R}}^{+}=(\max\mathscr{r}_{\imath},\min\mathscr{a}_{\imath},\min\mathscr{b}_{\imath})$. Then

$$\acute{\mathrm{R}}_{\imath}^{-}\le \mathrm{PFFMSM}\left(\acute{\mathrm{R}}_1,\acute{\mathrm{R}}_2,\acute{\mathrm{R}}_3,\cdots\acute{\mathrm{R}}_n\right)\le\ \acute{\mathrm{R}}_{\imath}^{+}.$$

Proof: Since $\mathscr{r}^{-}=\min(\mathscr{r}_{\imath})$, $\mathscr{a}^{+}=\max(\mathscr{a}_{\imath})$ and $\mathscr{b}^{+}=\max(\mathscr{b}_{\imath})$. So, we write as $\mathscr{r}^{-}\le\mathscr{r}_{\imath}\le\mathscr{r}^{+}$, $\mathscr{a}^{-}\le\mathscr{a}_{\imath}\le\ \mathscr{a}^{+}$, $\mathscr{b}^{-}\le\mathscr{b}_{\imath}\le\ \mathscr{b}^{+}$ for all . By the Theorem 3,4, we can write as

$$\acute{\mathrm{R}}_{\imath}^{-}\le \mathrm{PFFMSM}\left(\acute{\mathrm{R}}_1,\acute{\mathrm{R}}_2,\acute{\mathrm{R}}_3,\cdots\acute{\mathrm{R}}_n\right)\le\ \acute{\mathrm{R}}_{\imath}^{+}.$$

So the proof can be completed.

4. Picture Fuzzy Frank weighted Maclaurin Symmetric Mean operator

In Segment 3, the attributes are aggregates. In any case, it can't think about the significance of different aggregated attributes. Hence, we can PFFPMSM operator to beat the issue. We can involve R is give the weight where $B=\{\alpha_{\imath}\}$, $\imath=1,2,\cdots n$ and $\sum_{\imath=1}^{n}\alpha_{\imath}=1$ next, we can discuss it unless otherwise stated.

Definition 8. Let $\acute{\mathrm{R}}_{\imath}$ are an assortment of the PFVs. Then PFFWMSM: $\ddot{\mathcal{J}}^{n} to \ddot{\mathcal{J}}$ the operator is defined as and $\acute{\mathrm{R}}$ are weight vector

$$\mathrm{PFFWMSM}\left(\acute{\mathrm{R}}_1,\acute{\mathrm{R}}_2,\acute{\mathrm{R}}_3,\cdots\acute{\mathrm{R}}_n\right)=\left(\frac{\bigoplus_{\mathrm{F}\,1\le\imath_1<\imath_2<\imath_3<\cdots<\imath_{\varrho}\le n}\bigotimes_{\mathrm{F}\,j=1}^{\varrho}\alpha_{\imath_j}\acute{\mathrm{R}}_{\imath_j}}{\mathcal{C}_{\varrho}^{\mathrm{n}}}\right)^{1/\varrho} \tag{9}$$

Where, $\mathcal{C}_{\varrho}^{\mathrm{n}}$ is the binomial coefficient, $1\le\mathrm{i}_1<\mathrm{i}_2<\mathrm{i}_3<\cdots<\mathrm{i}_{\varrho}\le\mathrm{n}$ represents $\varrho-$tuple the PFVs.

The Eq. 4, in which we prove the fundamental PFFWMSM operator. We can aggregate the PFFWMSM in Theorem 1.

Theorem 5. The $\acute{R}_\iota = (r_\iota, a_\iota, b_\iota)$ be the assortment of PFVs. Then aggregated the value of PFS in the PFFWMSM operator,

$$\mathrm{PFFWMSM}\left(\acute{R}_1, \acute{R}_2, \acute{R}_3, \cdots \acute{R}_n\right) = \left(\begin{array}{c} \mathrm{Log}_\alpha\left(1 + \dfrac{\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\frac{\left(\alpha^{1-\mathcal{E}}-1\right)^{\mathcal{C}_\varrho^n}}{(\alpha-1)^{\mathcal{C}_\varrho^n-1}}\right)}-1\right)^{\frac{1}{\varrho}}}{(\alpha-1)^{\varrho-1}}\right), \\ 1-\mathrm{Log}_\alpha\left(1 + \dfrac{\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\frac{\left(\alpha^{a}-1\right)^{\mathcal{C}_\varrho^n}}{(\alpha-1)^{\mathcal{C}_\varrho^n-1}}\right)}-1\right)^{\frac{1}{\varrho}}}{(\alpha-1)^{\frac{1}{\varrho}-1}}\right), \\ 1-\mathrm{Log}_\alpha\left(1 + \dfrac{\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\frac{\left(\alpha^{\varphi}-1\right)^{\mathcal{C}_\varrho^n}}{(\alpha-1)^{\mathcal{C}_\varrho^n-1}}\right)}-1\right)^{\frac{1}{\varrho}}}{(\alpha-1)^{\frac{1}{\varrho}-1}}\right) \end{array} \right)$$

We can expect the pupations

$$\mathcal{E} = 1 - \mathrm{Log}_\alpha\left(1 + \prod_{1\le\iota_1<\iota_2<\iota_3<\cdots<\iota_\varrho\le n}\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\left(\prod_{\iota=1}^{n}\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\left(\alpha^{1-x_\iota}-1\right)^{\frac{1-\alpha_{\iota_j}}{\alpha_{\iota_j}}}(\alpha-1)^{1-\alpha_{\iota_j}}\right)}-1\right)\right)(\alpha-1)^{-1}\right)} - 1\right)(\alpha-1)^{1-\varrho}\right)$$

and

$$a = \mathrm{Log}_\alpha\left(1 + \prod_{1\le\iota_1<\iota_2<\iota_3<\cdots<\iota_\varrho\le n}\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\left(\prod_{\iota=1}^{n}\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\left(\alpha^{a_\iota}-1\right)^{\frac{1-\alpha_{\iota_j}}{\alpha_{\iota_j}}}(\alpha-1)^{1-\alpha_{\iota_j}}\right)}-1\right)\right)(\alpha-1)^{-1}\right)} - 1\right)(\alpha-1)^{1-\varrho}\right)$$

$$\varphi = \mathrm{Log}_{\alpha}\left(1 + \prod_{1 \le \imath_1 < \imath_2 < \imath_3 < \cdots < \imath_{\varrho} \le n}\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\left(\prod_{\imath=1}^{n}\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{\mathscr{b}_{\imath}}-1\right)^{\frac{1-\alpha_{\imath_j}}{\alpha_{\imath_j}}}(\alpha-1)^{1-\alpha_{\imath_j}}\right)}-1\right)\right)(\alpha-1)^{-1}\right)} - 1\right)(\alpha-1)^{1-\varrho}\right)$$

Proof: We can prove $\underset{j=1}{\overset{\varrho}{\otimes}}{}_{\mathrm{F}}\alpha_{\imath_j}\acute{\mathrm{R}}_{\imath_j}$ first, as follows,

$$\underset{j=1}{\overset{\varrho}{\otimes}}{}_{\mathrm{F}}\alpha_{\imath_j}\acute{\mathrm{R}}_{\imath_j} = \begin{pmatrix} \mathrm{Log}_{\alpha}\left(1+\left(\prod_{\imath=1}^{n}\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{1-r_{\imath}}-1\right)^{\alpha_{\imath_j}}(\alpha-1)^{1-\alpha_{\imath_j}}\right)}-1\right)\right)(\alpha-1)^{-1}\right), \\ 1-\mathrm{Log}_{\alpha}\left(1+\left(\prod_{\imath=1}^{n}\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{a_{\imath}}-1\right)^{\alpha_{\imath_j}}(\alpha-1)^{1-\alpha_{\imath_j}}\right)}-1\right)\right)(\alpha-1)^{-1}\right), \\ 1-\mathrm{Log}_{\alpha}\left(1+\left(\prod_{\imath=1}^{n}\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{\mathscr{b}_{\imath}}-1\right)^{\alpha_{\imath_j}}(\alpha-1)^{1-\alpha_{\imath_j}}\right)}-1\right)\right)(\alpha-1)^{-1}\right) \end{pmatrix}$$

Which is a PFV. Now, we calculate $\underset{1 \le \imath_1 < \imath_2 < \imath_3 < \cdots < \imath_{\varrho} \le n}{\oplus_{\mathrm{F}}}\underset{j=1}{\overset{\varrho}{\otimes}}{}_{\mathrm{F}}\alpha_{\imath_j}\acute{\mathrm{R}}_{\imath_j}$ as follows,

$$\underset{1 \le \imath_1 < \imath_2 < \imath_3 < \cdots < \imath_{\varrho} \le n}{\oplus_{\mathrm{F}}}\underset{j=1}{\overset{\varrho}{\otimes}}{}_{\mathrm{F}}\alpha_{\imath_j}\acute{\mathrm{R}}_{\imath_j}$$

$$= \begin{pmatrix} 1-\mathrm{Log}_{\alpha}\left(1 + \prod_{1 \le \imath_1 < \imath_2 < \imath_3 < \cdots < \imath_{\varrho} \le n}\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\left(\prod_{\imath=1}^{n}\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{1-r_{\imath}}-1\right)^{\frac{1-\alpha_{\imath_j}}{\alpha_{\imath_j}}}(\alpha-1)^{1-\alpha_{\imath_j}}\right)}-1\right)\right)(\alpha-1)^{-1}\right)} - 1\right)(\alpha-1)^{1-\varrho}\right), \\ \mathrm{Log}_{\alpha}\left(1 + \prod_{1 \le \imath_1 < \imath_2 < \imath_3 < \cdots < \imath_{\varrho} \le n}\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\left(\prod_{\imath=1}^{n}\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{a_{\imath}}-1\right)^{\frac{1-\alpha_{\imath_j}}{\alpha_{\imath_j}}}(\alpha-1)^{1-\alpha_{\imath_j}}\right)}-1\right)\right)(\alpha-1)^{-1}\right)} - 1\right)(\alpha-1)^{1-\varrho}\right), \\ \mathrm{Log}_{\alpha}\left(1 + \prod_{1 \le \imath_1 < \imath_2 < \imath_3 < \cdots < \imath_{\varrho} \le n}\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\left(\prod_{\imath=1}^{n}\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{\mathscr{b}_{\imath}}-1\right)^{\frac{1-\alpha_{\imath_j}}{\alpha_{\imath_j}}}(\alpha-1)^{1-\alpha_{\imath_j}}\right)}-1\right)\right)(\alpha-1)^{-1}\right)} - 1\right)(\alpha-1)^{1-\varrho}\right) \end{pmatrix}$$

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$$\mathcal{E}=1-\mathrm{Log}_{\alpha}\left(1+\prod_{1\leq\imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\leq n}\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\left(\prod_{\imath=1}^{n}\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{1-r_{\imath}}-1\right)^{\frac{1-\alpha_{\imath_j}}{\alpha_{\imath_j}}}(\alpha-1)^{1-\alpha_{\imath_j}}\right)}-1\right)\right)(\alpha-1)^{-1}\right)}-1\right)(\alpha-1)^{1-\varrho}\right)$$

and

$$a=\mathrm{Log}_{\alpha}\left(1+\prod_{1\leq\imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\leq n}\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\left(\prod_{\imath=1}^{n}\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{a_{\imath}}-1\right)^{\frac{1-\alpha_{\imath_j}}{\alpha_{\imath_j}}}(\alpha-1)^{1-\alpha_{\imath_j}}\right)}-1\right)\right)(\alpha-1)^{-1}\right)}-1\right)(\alpha-1)^{1-\varrho}\right)$$

$$\varphi=\mathrm{Log}_{\alpha}\left(1+\prod_{1\leq\imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\leq n}\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\left(\prod_{\imath=1}^{n}\left(\alpha^{1-\mathrm{Log}_{\alpha}\left(1+\left(\alpha^{b_{\imath}}-1\right)^{\frac{1-\alpha_{\imath_j}}{\alpha_{\imath_j}}}(\alpha-1)^{1-\alpha_{\imath_j}}\right)}-1\right)\right)(\alpha-1)^{-1}\right)}-1\right)(\alpha-1)^{1-\varrho}\right)$$

Further we have

$$\frac{\bigoplus_{\mathrm{F}\,1\leq\imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\leq n}\bigotimes_{\mathrm{F}\,j=1}^{\varrho}\alpha_{\imath_j}\acute{\mathrm{R}}_{\imath_j}}{\mathcal{C}_{\varrho}^{n}}=\left(\begin{array}{c}1-\mathrm{Log}_{\alpha}\left(1+\dfrac{(\alpha^{1-\mathcal{E}}-1)^{\mathcal{C}_{\varrho}^{n}}}{(\alpha-1)^{\mathcal{C}_{\varrho}^{n}-1}}\right),\\ \mathrm{Log}_{\alpha}\left(1+\dfrac{(\alpha^{a}-1)^{\mathcal{C}_{\varrho}^{n}}}{(\alpha-1)^{\mathcal{C}_{\varrho}^{n}-1}}\right),\mathrm{Log}_{\alpha}\left(1+\dfrac{(\alpha^{\varphi}-1)^{\mathcal{C}_{\varrho}^{n}}}{(\alpha-1)^{\mathcal{C}_{\varrho}^{n}-1}}\right)\end{array}\right)$$

Finally,

$$\left(\frac{\bigoplus_{F\, 1\le \imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n} \bigotimes_{F\, j=1}^{\varrho} \alpha_{\imath_j} \acute{R}_{\imath_j}}{\mathcal{C}_\varrho^n}\right)^{\frac{1}{\varrho}} = \begin{pmatrix} \mathrm{Log}_\alpha\left(1+\frac{\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\frac{\left(\alpha^{1-\mathcal{E}}-1\right)^{\mathcal{C}_\varrho^n}}{(\alpha-1)^{\mathcal{C}_\varrho^n-1}}\right)}-1\right)^{\frac{1}{\varrho}}}{(\alpha-1)^{\varrho-1}}\right), \\ 1-\mathrm{Log}_\alpha\left(1+\frac{\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\frac{\left(\alpha^{a}-1\right)^{\mathcal{C}_\varrho^n}}{(\alpha-1)^{\mathcal{C}_\varrho^n-1}}\right)}-1\right)^{\frac{1}{\varrho}}}{(\alpha-1)^{\frac{1}{\varrho}-1}}\right), \\ 1-\mathrm{Log}_\alpha\left(1+\frac{\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\frac{\left(\alpha^{\varphi}-1\right)^{\mathcal{C}_\varrho^n}}{(\alpha-1)^{\mathcal{C}_\varrho^n-1}}\right)}-1\right)^{\frac{1}{\varrho}}}{(\alpha-1)^{\frac{1}{\varrho}-1}}\right) \end{pmatrix}$$

The theorem can be completed.

Theorem 6 (Idempotency). Let $\acute{R}_\imath = (r_\imath, a_\imath, b_\imath)$ be the three of PFVs, and if $\acute{R}_\imath = (r_\imath, a_\imath, b_\imath) = (r, a, b) = \acute{R}\ \forall\ (\imath = 1,2,3,\cdots,n)$. Then $\mathrm{PFFWMSM}(\acute{R}_1, \acute{R}_2, \acute{R}_3, \cdots \acute{R}_n) = \acute{R}$ is called the idempotency.

Proof: As $\acute{R}_\imath = (r_\imath, a_\imath, b_\imath) = \acute{R}\ \forall\ (\imath = 1,2,3,\cdots,n)$. Then we have

$$\bigotimes_{F\, j=1}^{\varrho} \alpha_{\imath_j} \acute{R}_{\imath_j} = \begin{pmatrix} \mathrm{Log}_\alpha\left(1+\left(\prod_{\imath=1}^{n}\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\left(\alpha^{1-r_\imath}-1\right)^{\alpha_{\imath_j}}(\alpha-1)^{1-\alpha_{\imath_j}}\right)}-1\right)\right)(\alpha-1)^{-1}\right), \\ 1-\mathrm{Log}_\alpha\left(1+\left(\prod_{\imath=1}^{n}\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\left(\alpha^{a_\imath}-1\right)^{\alpha_{\imath_j}}(\alpha-1)^{1-\alpha_{\imath_j}}\right)}-1\right)\right)(\alpha-1)^{-1}\right), \\ 1-\mathrm{Log}_\alpha\left(1+\left(\prod_{\imath=1}^{n}\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\left(\alpha^{b_\imath}-1\right)^{\alpha_{\imath_j}}(\alpha-1)^{1-\alpha_{\imath_j}}\right)}-1\right)\right)(\alpha-1)^{-1}\right) \end{pmatrix}$$

$$= (r, a, b)$$

Further,

$$\bigoplus_{F\,1\le \imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\ \bigotimes_{F\,\jmath=1}^{\varrho}\alpha_{\imath_\jmath}\acute{R}_{\imath_\jmath}$$

$$= \left(\begin{array}{l} 1-\mathrm{Log}_\alpha\left(1+\prod_{1\le \imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\left(\prod_{\imath=1}^{n}\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\left(\alpha^{1-r_\imath}-1\right)^{\frac{1-\alpha_{\imath_\jmath}}{\alpha_{\imath_\jmath}}}(\alpha-1)^{1-\alpha_{\imath_\jmath}}\right)}-1\right)\right)(\alpha-1)^{-1}\right)}-1\right)(\alpha-1)^{1-\varrho}\right), \\ \mathrm{Log}_\alpha\left(1+\prod_{1\le \imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\left(\prod_{\imath=1}^{n}\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\left(\alpha^{a_\imath}-1\right)^{\frac{1-\alpha_{\imath_\jmath}}{\alpha_{\imath_\jmath}}}(\alpha-1)^{1-\alpha_{\imath_\jmath}}\right)}-1\right)\right)(\alpha-1)^{-1}\right)}-1\right)(\alpha-1)^{1-\varrho}\right), \\ \mathrm{Log}_\alpha\left(1+\prod_{1\le \imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\left(\prod_{\imath=1}^{n}\left(\alpha^{1-\mathrm{Log}_\alpha\left(1+\left(\alpha^{b_\imath}-1\right)^{\frac{1-\alpha_{\imath_\jmath}}{\alpha_{\imath_\jmath}}}(\alpha-1)^{1-\alpha_{\imath_\jmath}}\right)}-1\right)\right)(\alpha-1)^{-1}\right)}-1\right)(\alpha-1)^{1-\varrho}\right) \end{array} \right)$$

$$= (r, a, b)$$

Finally,

$$\left(\frac{\bigoplus_{F\,1\le \imath_1<\imath_2<\imath_3<\cdots<\imath_\varrho\le n}\ \bigotimes_{F\,\jmath=1}^{\varrho}\alpha_{\imath_\jmath}\acute{R}_{\imath_\jmath}}{C_\varrho^n}\right)^{\frac{1}{\varrho}} = (r, b)$$

The theorem can be completed.

Theorem 7 (Monotonicity). Let $\acute{R}_\imath = (r_\imath, a_\imath, b_\imath)$ and $\acute{R}_\imath^a = (r_\imath^a, a_\imath^a, b_\imath^a)$ be the three of PFVs, and if $\acute{R}_\imath \ge \acute{R}_\imath^a$ for all then

$$\mathrm{PFFWMSM}(\acute{R}_1, \acute{R}_2, \acute{R}_3, \cdots \acute{R}_n) \ge \mathrm{PFFWMSM}(\acute{R}_1^a, \acute{R}_2^a, \acute{R}_3^a, \cdots \acute{R}_n^a).$$

Proof: As stated that $\acute{R}_\imath \ge \acute{R}_\imath^a$ so the PFVs $r_\imath \ge r_\imath^a$ $a_\imath \le a_\imath^a$ and $b_\imath \le b_\imath^a$. We have

$$\bigotimes_{F\,\jmath=1}^{\varrho}\alpha_{\imath_\jmath}\acute{R}_{\imath_\jmath} \ge \bigotimes_{F\,\jmath=1}^{\varrho}\alpha_{\imath_\jmath}\acute{R}_{\imath_\jmath}^a$$

Further,

$$\frac{\underset{1\le\iota_1<\iota_2<\iota_3<\cdots<\iota_\varrho\le n}{\oplus_F} \overset{\varrho}{\underset{j=1}{\otimes_F}} \alpha_{\iota_j} \acute{R}_{\iota_j}}{\mathcal{C}_\varrho^n} \ge \frac{\underset{1\le\iota_1<\iota_2<\iota_3<\cdots<\iota_\varrho\le n}{\oplus_F} \overset{\varrho}{\underset{j=1}{\otimes_F}} \alpha_{\iota_j} \acute{R}^a_{\iota_j}}{\mathcal{C}_\varrho^n}$$

Finally,

$$\left(\frac{\underset{1\le\iota_1<\iota_2<\iota_3<\cdots<\iota_\varrho\le n}{\oplus_F} \overset{\varrho}{\underset{j=1}{\otimes_F}} \alpha_{\iota_j} \acute{R}_{\iota_j}}{\mathcal{C}_\varrho^n}\right)^{\frac{1}{\varrho}} \ge \left(\frac{\underset{1\le\iota_1<\iota_2<\iota_3<\cdots<\iota_\varrho\le n}{\oplus_F} \overset{\varrho}{\underset{j=1}{\otimes_F}} \alpha_{\iota_j} \acute{R}^a_{\iota_j}}{\mathcal{C}_\varrho^n}\right)^{\frac{1}{\varrho}}$$

The theorem can be completed.

Theorem 8 (Boundedness). Let $\acute{R}_\iota = (r_\iota, a_\iota, b_\iota)$ be the three of PFVs, if $\acute{R}^- = \min(\acute{R}_\iota)$, $\acute{R}^+ = \max(\acute{R}_\iota)$, Then

$$\acute{R}_\iota^- \le \text{PFFWMSM}(\acute{R}_1, \acute{R}_2, \acute{R}_3, \cdots \acute{R}_n) \le \acute{R}_\iota^+$$

Proof: Since and $b^+ = \max(b_\iota)$. We can write $r^- \le r_\iota \le r^+, a^- \le a_\iota \le a^+, b^- \le b_\iota \le b^+$ for all $\iota = 1,2,3,\cdots,n$. According to the Theorem 6 and 7

$$\acute{R}_\iota^- \le \text{PFFWMSM}(\acute{R}_1, \acute{R}_2, \acute{R}_3, \cdots \acute{R}_n) \le \acute{R}_\iota^+.$$

The theorem can be completed.

5. The Application Approach to the MAGDM

This section incorporates the system application PFFWMSM operator, the dynamic issues, and the data of fuzziness as PFVs. In this section, we presented the application of the proposed methodology in the organization and the improvement of the business field. This part of the PFFPMSM overseer is differentiated, and ongoing AOs are the importance of AOs. The alternatives of n alternatives $Z = \{\zeta_1, \zeta_2, \ldots, \zeta_n\}$ from are someone alternative has must be $\mathcal{C}$hosen on m attributes $B = \{\beta_1, \beta_2, \ldots, \beta_m\}$ having the weights from $\Psi = \{\psi_1, \psi_2, \ldots, \psi_m\}$ with $\sum_{\iota=1}^m \psi_\iota \ \forall \iota = 1,2,3,\cdots,m$. Let $r = \{\kappa_1, \kappa_2, \ldots, \kappa_j\}$ be the weights of j specialists and $T^s = \left[r^s{}_{\iota j}\right]_{m\times n}$ be the decision matrix where $r^s{}_{\iota j} = \left(r^s_{\iota j}, a^s_{\iota j}, b^s_{\iota j}\right)$ be the PFV information. The decision-making of the PFFPMSM operator is detailed below.

Step 1. PFV information can be normalized in the form.

There are two properties all around, i.e., cost and benefit. The different impact of characteristics ought to be the same aggregate as before. The below data can be normalized as follows:

$$(T^s)^c = \begin{cases} \left(r^s_{\iota j}, a^s_{\iota j}, b^s_{\iota j}\right) & \text{for benefit attribute} \\ \left(b^s_{\iota j}, a^s_{\iota j}, r^s_{\iota j}\right) & \text{for cost attribute} \end{cases}$$

Step 2. The PFS information is proposed PFFϱMSM, Eq. (9), to get the particular tendencies values of decision networks.

Step 3. The PFFWMSM can apply Eq. (8) to get total tendency Qualities.

Step 4. With the help of Eq. (1), we can find the value of the score.

Step 5. Position of other options.

In the case study, the turn of events influences business advancement to numerous perils. Some huge models are political uncertainty, poor money-related systems, lamentable trade procedures, startling briefs, etc. The difficult issue is that there is no assessment device to measure the effects of these components; in this way, we can't assess the fundamental thought so that we can push toward a decline in the effects of that part. In addition, each part can be established on specific characteristics. Subsequently, we can apply the PFFWMSM executive to evaluate the variable expected to have been in general on need by dealing with a MAGDM issue in the model 1 accompanying.

Example. 5.1. Advancement business is the backbone of any country's economy. The high-level countries are making incredible advancements in the industry since the base on procedures is made for improvement. In any case, improvement business depends on the specific moreover and the attributes there are some bet factors for it. With this model, we can examine the problem of the board's bet: the components ought to have broken down and, for the most part, remain identifiable. Let $1 \leq \mathcal{J}_\iota \leq 4$ are seen the poor monetary procedures, weakness of political, appalling method log y, and any advancement industry hidden separating light of specific qualities $1 \leq \vartheta_i \leq 4$ weights $(0.2,0.3,0.25,0.25)$. Three experts are surveying bet factors that are considered recently referenced. The qualities doled out to every option from the choice specialists as the PFVs in Table 1-3.

Table 1
Picture fuzzy Decision matrix D1

	$\mathcal{J}_1$			$\mathcal{J}_2$			$\mathcal{J}_3$			$\mathcal{J}_4$		
	$\mathscr{r}$	$\mathscr{a}$	$\mathscr{b}$	$\mathscr{r}$	$\mathscr{a}$	$\mathscr{b}$	$\mathscr{r}$	$\mathscr{a}$	$\mathscr{b}$	$\mathscr{r}$	$\mathscr{a}$	$\mathscr{b}$
ϑ_1	0.1	0.3	0.2	0.1	0.2	0.3	0.3	0.1	0.2	0.4	0.3	0.2
ϑ_2	0.4	0.1	0.2	0.4	0.3	0.2	0.4	0.3	0.2	0.3	0.1	0.4
ϑ_3	0.3	0.2	0.1	0.2	0.4	0.3	0.2	0.4	0.2	0.3	0.4	0.1
ϑ_4	0.2	0.4	0.3	0.4	0.1	0.2	0.4	0.2	0.1	0.2	0.3	0.1

Table 2
Picture fuzzy Decision matrix D2

	$\mathcal{J}_1$			$\mathcal{J}_2$			$\mathcal{J}_3$			$\mathcal{J}_4$		
	$\mathscr{r}$	$\mathscr{a}$	$\mathscr{b}$	$\mathscr{r}$	$\mathscr{a}$	$\mathscr{b}$	$\mathscr{r}$	$\mathscr{a}$	$\mathscr{b}$	$\mathscr{r}$	$\mathscr{a}$	$\mathscr{b}$
ϑ_1	0.2	0.3	0.3	0.4	0.3	0.1	0.4	0.3	0.1	0.4	0.1	0.2
ϑ_2	0.3	0.2	0.1	0.3	0.1	0.2	0.3	0.4	0.2	0.3	0.2	0.3
ϑ_3	0.2	0.3	0.2	0.3	0.4	0.1	0.1	0.4	0.3	0.1	0.4	0.2
ϑ_4	0.1	0.4	0.1	0.3	0.4	0.2	0.2	0.3	0.4	0.4	0.1	0.4

Table 3
Picture fuzzy Decision matrix D3

	$\mathcal{J}_1$			$\mathcal{J}_2$			$\mathcal{J}_3$			$\mathcal{J}_4$		
	$\mathscr{r}$	$\mathscr{a}$	$\mathscr{b}$	$\mathscr{r}$	$\mathscr{a}$	$\mathscr{b}$	$\mathscr{r}$	$\mathscr{a}$	$\mathscr{b}$	$\mathscr{r}$	$\mathscr{a}$	$\mathscr{b}$
ϑ_1	0.2	0.5	0.3	0.3	0.1	0.2	0.2	0.4	0.1	0.3	0.2	0.1
ϑ_2	0.2	0.3	0.1	0.4	0.2	0.3	0.4	0.3	0.2	0.1	0.4	0.2
ϑ_3	0.1	0.5	0.1	0.3	0.3	0.2	0.1	0.4	0.3	0.2	0.3	0.3
ϑ_4	0.3	0.3	0.2	0.4	0.3	0.1	0.4	0.2	0.1	0.3	0.4	0.1

Step 1. In the above data, every one of the qualities is similar. It comes quaintly, and we needn't bother with the standardization since the properties are expense characteristics.

Step 2. We smear the PFFWMSM operator to total all the upsides of qualities in the given 1- 3. Given the value in Table 4 as follows,

Table 4
Collective Preference Values obtained by PFFWMSM

	$\mathcal{J}_1$			$\mathcal{J}_2$			$\mathcal{J}_3$			$\mathcal{J}_4$		
	r	a	b	r	a	b	r	a	b	r	a	b
ϑ_1	0.40	0.60	0.57	0.42	0.36	0.30	0.67	0.62	0.10	0.53	0.37	0.27
ϑ_2	0.54	0.43	0.64	0.52	0.36	0.31	0.74	0.71	0.11	0.41	0.40	0.33
ϑ_3	0.43	0.57	0.64	0.44	0.52	0.30	0.44	0.77	0.13	0.37	0.53	0.29
ϑ_4	0.43	0.61	0.61	0.52	0.42	0.29	0.70	0.60	0.15	0.48	0.43	0.29

Step 3: We can PFFWMSM operator in Table obtain the individual results in Table 5,

Table 5
Individual preferences result in Table 4

	$\mathcal{J}_1$			$\mathcal{J}_2$			$\mathcal{J}_3$			$\mathcal{J}_4$		
	r	a	b	r	a	b	r	a	b	r	a	b
r_{ij}	0.715	0.805	0.004	0.738	0.683	0.072	0.871	0.894	0.238	0.712	0.700	0.076

Step 4. We can obtain the rank of the result by using the score function in Table 6.

Table 6
Score values

$sce(\mathcal{J}_1)$	$sce(\mathcal{J}_2)$		$sce(\mathcal{J}_3)$	$sce(\mathcal{J}_4)$
-0.094	-0.017		-0.260	-0.064

As we obtain $sce(\mathcal{J}_1) > sce(\mathcal{J}_3) > sce(\mathcal{J}_4) > sce(\mathcal{J}_2)$, hence $\mathcal{J}_1 \succ \mathcal{J}_3 \succ \mathcal{J}_4 \succ \mathcal{J}_2$, which means that $\mathcal{J}_4$ is the most; this means that $\mathcal{J}_1$ the alternative is the most appropriate.

5.1. Influence Study

The basic attribute of FTN and TCN is that these tasks are adaptable because of the limit α**.** Nonetheless, the outcomes could appear differently concerning the worth of this breaking point. In the following section, we will see the groupings of the outcomes in Table 7.

Table 7
PFFwMSM operator Variation with different parameter

α	Ranking of score values	Preference Orders
2	$sce(\Gamma_4) > sce(\mathcal{J}_2) > sce(\mathcal{J}_1) > sce(\mathcal{J}_3)$	$\mathcal{J}_4 \succ \mathcal{J}_2 \succ \mathcal{J}_1 \succ \mathcal{J}_3$
3	$sce(\mathcal{J}_1) > sce(\mathcal{J}_2) > sce(\mathcal{J}_4) > sce(\mathcal{J}_3)$	$\mathcal{J}_1 \succ \mathcal{J}_2 \succ \mathcal{J}_4 \succ \mathcal{J}_3$
4	$sce(\mathcal{J}_1) > sce(\mathcal{J}_2) > sce(\mathcal{J}_4) > sce(\mathcal{J}_3)$	$\mathcal{J}_1 \succ \mathcal{J}_2 \succ \mathcal{J}_4 \succ \mathcal{J}_3$
5	$sce(\mathcal{J}_1) > sce(\mathcal{J}_2) > sce(\mathcal{J}_4) > sce(\mathcal{J}_3)$	$\mathcal{J}_1 \succ \mathcal{J}_2 \succ \mathcal{J}_4 \succ \mathcal{J}_3$
6	$sce(\mathcal{J}_1) > sce(\mathcal{J}_2) > sce(\mathcal{J}_4) > sce(\mathcal{J}_3)$	$\mathcal{J}_1 \succ \mathcal{J}_2 \succ \mathcal{J}_4 \succ \mathcal{J}_3$
7	$sce(\mathcal{J}_1) > sce(\mathcal{J}_2) > sce(\mathcal{J}_4) > sce(\mathcal{J}_3)$	$\mathcal{J}_1 \succ \mathcal{J}_2 \succ \mathcal{J}_4 \succ \mathcal{J}_3$
8	$sce(\mathcal{J}_1) > sce(\mathcal{J}_2) > sce(\mathcal{J}_4) > sce(\mathcal{J}_3)$	$\mathcal{J}_1 \succ \mathcal{J}_2 \succ \mathcal{J}_4 \succ \mathcal{J}_3$
9	$sce(\mathcal{J}_1) > sce(\mathcal{J}_2) > sce(\mathcal{J}_4) > sce(\mathcal{J}_3)$	$\mathcal{J}_1 \succ \mathcal{J}_2 \succ \mathcal{J}_4 \succ \mathcal{J}_3$
10	$sce(\mathcal{J}_2) > sce(\mathcal{J}_4) > sce(\mathcal{J}_1) > sce(\mathcal{J}_3)$	$\mathcal{J}_2 \succ \mathcal{J}_4 \succ \mathcal{J}_1 \succ \mathcal{J}_3$
15	$sce(\mathcal{J}_1) > sce(\mathcal{J}_2) > sce(\mathcal{J}_4) > sce(\mathcal{J}_3)$	$\mathcal{J}_1 \succ \mathcal{J}_2 \succ \mathcal{J}_4 \succ \mathcal{J}_3$

α	Ranking of score values	Preference Orders
20	$sce(\mathcal{J}_1) > sce(\mathcal{J}_2) > sce(\mathcal{J}_4) > sce(\mathcal{J}_3)$	$\mathcal{J}_1 \succ \mathcal{J}_2 \succ \mathcal{J}_4 \succ \mathcal{J}_3$
30	$sce(\mathcal{J}_1) > sce(\mathcal{J}_2) > sce(\mathcal{J}_4) > sce(\mathcal{J}_3)$	$\mathcal{J}_1 \succ \mathcal{J}_2 \succ \mathcal{J}_4 \succ \mathcal{J}_3$
40	$sce(\mathcal{J}_1) > sce(\mathcal{J}_2) > sce(\mathcal{J}_4) > sce(\mathcal{J}_3)$	$\mathcal{J}_1 \succ \mathcal{J}_2 \succ \mathcal{J}_4 \succ \mathcal{J}_3$
50	$sce(\mathcal{J}_1) > sce(\mathcal{J}_2) > sce(\mathcal{J}_4) > sce(\mathcal{J}_3)$	$\mathcal{J}_1 \succ \mathcal{J}_2 \succ \mathcal{J}_4 \succ \mathcal{J}_3$

We have investigated the results gained by PFFWMSM executives by vitiating the potential gains of α. We can take the α value from 2 to 50. The characterized results are in Table 7. All of α the potential gains can be found in the monetary courses of action as the critical bet factor for the advancement of business. PFFWMSM overseer is consistent after all of the potential gains of $\alpha \geq 2$, and the chief can decide to select the worth of α we can find the primary property of the FTN and TCM. We can recommend the $\alpha \geq 2$ best results. Results from Table 7 are presented in Figure 1.

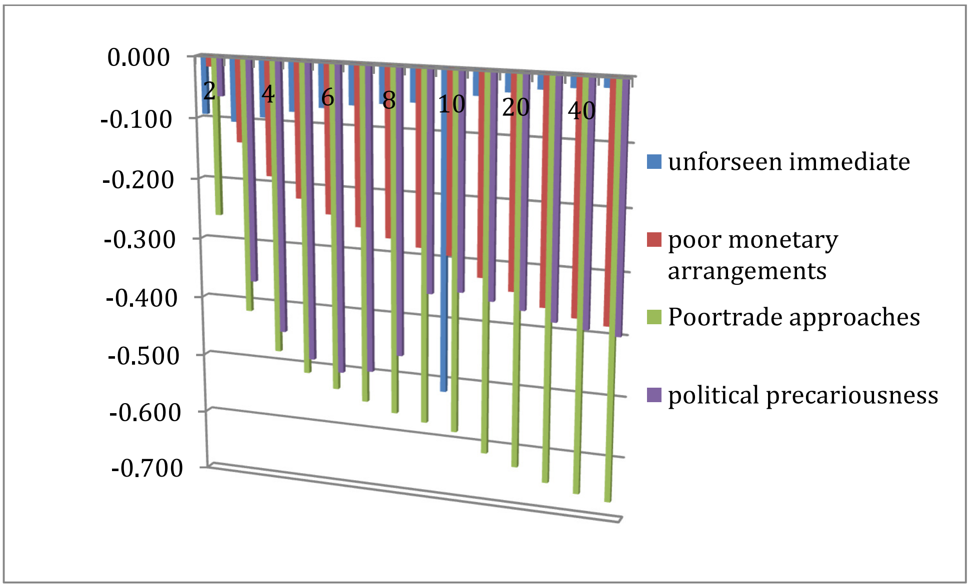

Fig. 1. Different score values given the same result parameter α from Table 7

The score values have different values of the parameter α. Figure 1 can be clear, creating the impression that the score values are the same on the different parameters α.

6. Comparative Study

This section aims to compare the recently developed MSM operators and other current AOs of PFVs. PFVs are an advanced frame that allows for expressing uncertain data within a specific range with less information loss. The last few years have seen the development of a large number of AOs. To assess the viability of the suggested MSM operators, our goal in this case is to compare the MSM operators with other AOs of PFs numerically. We apply the picture fuzzy MSM operator by Ullah [40], picture fuzzy Archimedean power MSM operator by Qin *et al.,* [19], MAGDM based on Hesitant PF Schweizer-Sklar by Chen and Ye [23], PF cross entropy for MADM problem by Wei [41], PF theories and methodologies by Peng *et al.*, [42] and MSM Schweizer-Sklar operator by Dong Wei Liu *et al.,* [43]. The outcomes are displayed in Table 8.

Table 8
Comparison of PFFWMSM operators

Operator	Ranking of score values	Preference Orders
PFFWMSM	$sce(\mathcal{J}_1) > sce(\mathcal{J}_3) > sce(\mathcal{J}_4) > sce(\mathcal{J}_2)$	$\mathcal{J}_1 \succ \mathcal{J}_3 \succ \mathcal{J}_4 \succ \mathcal{J}_2$
Ullah [40]	$sce(\mathcal{J}_1) > sce(\mathcal{J}_3) > sce(\mathcal{J}_4) > sce(\mathcal{J}_2)$	$\mathcal{J}_1 \succ \mathcal{J}_3 \succ \mathcal{J}_4 \succ \mathcal{J}_2$
Qin *et al.*, [19]	$sce(\mathcal{J}_1) > sce(\mathcal{J}_3) > sce(\mathcal{J}_4) > sce(\mathcal{J}_2)$	$\mathcal{J}_1 \succ \mathcal{J}_3 \succ \mathcal{J}_4 \succ \mathcal{J}_2$
Chen and Ye [23]	$sce(\mathcal{J}_1) > sce(\mathcal{J}_3) > sce(\mathcal{J}_4) > sce(\mathcal{J}_2)$	$\mathcal{J}_1 \succ \mathcal{J}_3 \succ \mathcal{J}_4 \succ \mathcal{J}_2$
Liu *et al.*, [43]	$sce(\mathcal{J}_1) > sce(\mathcal{J}_3) > sce(\mathcal{J}_4) > sce(\mathcal{J}_2)$	$\mathcal{J}_1 \succ \mathcal{J}_3 \succ \mathcal{J}_4 \succ \mathcal{J}_2$
Peng *et al.*, [42]	$sce(\mathcal{J}_1) > sce(\mathcal{J}_3) > sce(\mathcal{J}_4) > sce(\mathcal{J}_2)$	$\mathcal{J}_1 \succ \mathcal{J}_3 \succ \mathcal{J}_4 \succ \mathcal{J}_2$
Wei [41]	$sce(\mathcal{J}_1) > sce(\mathcal{J}_3) > sce(\mathcal{J}_4) > sce(\mathcal{J}_2)$	$\mathcal{J}_1 \succ \mathcal{J}_3 \succ \mathcal{J}_4 \succ \mathcal{J}_2$

Table 8 uses the different operators by ranking the risk factors in order. Results show that alternative $\mathcal{J}_1$ presents a dominant alternative from the considered sets. Since the comparative result presents the same rankings, we can conclude that the proposed mode has been validated.

5. Conclusion

First, this paper introduces some fundamental PFV operations based on Frank TN and TCN. Next, we provided the AOs to compile the PF data. Then, we introduced the PFFMSM and PFFWMSM operators and showed their effectiveness in decision-making. Additionally, we applied the proposed mathematical model to a real-world example and presented sensitivity analysis by varying the values of the involved parameter α. The sensitivity analysis examined the behavior of these operators in more detail. The suggested operators were compared to PFFWMSM operators. The intriguing findings that followed were discovered: (1) First, because of Frank TN and TCN, the PFFWMSM operators are more versatile than the other comparable operators; (2) Secondly, the PFFWMSM operators rely on a single parameter α, which can be varied to provide a distinct ranking of options; (3) Third, we use the alternative's weighted average to determine which the superior choice is. In future studies, researchers should consider the application of the presented framework with the picture hesitant FS, complex picture hesitant FS, Spherical hesitant FS, and T-spherical hesitant FS.

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Conflicts of Interest
The authors declare that any known competing financial interests or personal relationships could have influenced none of the work reported in this study.

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