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Challenges for Adapting High-speed Rail Network in Developing Countries: A Picture Fuzzy MCDM based Approach

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ABSTRACT

High-speed rail adoption in developing countries faces challenges for several reasons. Our research work identifies the most impactful challenges of it. It is an extremely demanding work. It mainly depends on several factors, including high capital costs and financing issues, the purchase of land and resettlement, political instability that affects policy continuity, maintenance and operating costs, the current market value, technology and technical skills, and the impact on both the environment and human society. In this study, the proposed adoption challenges for high-speed rail networks in developing countries are constructed and processed using the Multi-Criteria Decision Making (MCDM) process in an uncertain environment. Here, we apply the Fuzzy Decision Making Trial and Evaluation Laboratory (DEMATEL) technique-based MCDM methodology to evaluate the criteria weights. We gather valuable opinions from three decision-makers (DMs) for data collection. Picture Fuzzy Numbers (PFNs) are utilised as uncertainty tools to address ambiguity.

1. Introduction

High-speed rail has revolutionized modern rapid transport, which is efficient and environmentally friendly. These trains run much faster than traditional rail, using a combination of special rolling stock and dedicated tracks. Lines that are elevated to speeds of 250 km/h for at least 155 miles or 200 km/h for 125 miles are generally considered high-speed. These trains are also known worldwide as bullet trains due to their streamlined spitzer-shaped nose angle. In 1964, the Tokaido Shinkansen began op-

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erating the first high-speed rail system in Honshu, Japan. Following them, Italy also introduced this system through the Direttissima line. Later, European countries such as France, Germany, and Spain followed this system and built high-speed rail systems. Despite being a developing country in the 21st century, China has played a leading role in high-speed railways, already occupying more than two-thirds of the world's railways by 2023. As China's high-speed rail network grows, developing countries are also expanding their use. Countries such as India, Turkey, Morocco, Malaysia (to Singapore), Egypt, Brazil, Russia, Kazakhstan, Iran, Thailand, Mexico, Indonesia, South Africa, and Vietnam are also planning to build this system. It is known that 24,602 (Km of line) projects are in operation, 11,807 (Km of line) are under construction, and 22,506 (Km of line) projects are still in long-term planning based on data from UIC (July 2017) [1]. And, in India, the 534 km Mumbai-Ahmedabad segment has been selected as the first route for High-speed rail [2].

1.1 Necessity for High-Speed Rail infrastructure

Due to various circumstances, the high-speed rail infrastructure has become increasingly necessary for modern transportation systems. This system significantly reduces travel distances, making it an attractive alternative to private cars and aeroplanes and is effective in reducing airport and overall traffic congestion. It also efficiently connects major cities, increasing trade and business opportunities and employment, helping boost economic activity and regional development. Compared to normal cars or planes, these trains emit significantly less carbon dioxide per kilometre, making them known as environmentally friendly transport. As they are powered by electricity, they are less dependent on fossil fuels, so their internal combustion is much more powerful and energy-efficient than that of ordinary vehicles. The high-speed rail infrastructure promotes public safety as rail transport has a lower accident rate than road transport. Also, with the increase in population and urbanization, this infrastructure becomes essential as an important transportation system for sustainable mobility.

1.2 Obstacles for Adapting High-Speed Rail Network in developing countries

There are several issues in building high-speed rail networks in developing countries, the main ones being high infrastructure costs, space issues, safety and reliability concerns, technology requirements, etc. The construction of this large-scale project requires massive investments in special tracks, station signals, power systems and rolling stock, which are beyond the budgets of developing countries. Legal problems for land acquisition are widespread in developing countries. This type of land acquisition often leads to legal disputes and delays in projects due to conflicts between the relevant communities. Also, due to the high population in developing countries, this construction can cause concerns about noise pollution and habitat loss, as well as damage to the natural ecosystem. Since it is a complex production and maintenance process, the lack of skilled workers and skilled technology can pose safety issues for ordinary passengers. In addition, political and institutional corruption or instability can create barriers in this regard.

1.3 Global developments and technological advancements for High-Speed Rail Network

High-speed railway networks around the world are showing advances in both conventional and modern technologies and help the global developments. The main goals of these advances are to increase speed, improve energy efficiency, and enhance safety and reliability. The expansion of this high-speed railway network across Asia and Europe has been so extensive that by 2024, China will lead the way with more than 45,000 kilometres of high-speed railway route [3]. Famous bullet trains like the China Railways SS8 (Shaoshan 8) electric locomotive, China Railways DJJ1 (Blue Arrow), China Railways DJF2 (Xianfeng), China Railways DJJ2 (China Star) are high-speed trains made in China [4]. Various international projects, such as the EU TEN-T (Trans-European Transport Network), Japan's Partnership for Quality Infrastructure (PQI), China's Belt and Road Initiative and the African Integrated High-Speed Rail Network (AIHSRN), encourage international connectivity. Private sector involvement has played a key role in this, with companies like Virgin Trains USA (Brightline) in the U.S. [5] demonstrating new financing strategies for privately funded high-speed transport system models.

Improved high-speed train design, advanced infrastructure and more operating efficiency are just a few of the technological advancements propelling the rail networks forward. The invention of the Maglev trains plays a significant role in this field. For example, Japan's Chuo Shinkansen train is expected to be built by 2027, with a speed of 600 kilometres per hour and very low noise and friction. Although the Hyperloop concept is still in its early stages, it will help accelerate high-speed systems in the future. The use of technologies such as artificial intelligence (AI), the Internet of Things (IoT) helps to improve safety, schedules and reduce downtime. Moreover, the use of solar-powered stations and hybrid-electric systems helps reduce carbon emissions.

1.4 *Motivation of this study*

Developing countries' high-speed railway networks are experiencing a need to modernize infrastructure to accommodate the changing economic landscape and increasing regional connectivity. This network provides a quicker, safer and more efficient alternative to traditional travel methods. It attracts significant investment and tourism. In addition, it helps reduce people's dependence on cars and planes and minimizes environmental pollution. This is, from the Government's perspective, a symbol of technological progress and patriotism. In this project, the adaptation challenges of adapting a high-speed rail network in developing countries are examined. For this real-life model-based work, the criteria for challenges of adaptation to a high-speed rail network are selected from various aspects, including demand, economy, geography, technology, environment, and governance. It displays a multi-criteria decision-making problem after the formulation of the model. Then, we process this using the DEMATEL technique in an uncertain environment.

1.5 *Novelties of this research*

The novelties of the study are clarified in this section. Adapting to high-speed rail networks in developing countries is a challenging issue. In this study, we consider several challenges to adapting a high-speed rail network in developing countries and explore & discuss. The feasibility of using this technology is taken into consideration. A new score function and accuracy function are proposed to determine score-valued discrete Picture Fuzzy Numbers (PFNs). A robust decision-making technique, namely DEMATEL, is employed to numerically analyze this concept and validate the already proposed discrete picture fuzzy framework. Furthermore, three wise decision-makers (DMs) assessments are incorporated into the decision-making process to guarantee impartial gathering of information, dependability and fairness. To effectively capture the ambiguity and vagueness inherent in the decision-making process, discrete picture fuzzy numbers (PFNs) are adopted in the proposed model.

1.6 *Structure of this paper*

This section goes into great depth on the total framework of the study. Section 1 introduces the introduction, including the necessity for High-Speed rail infrastructure, obstacles to adapting this network in developing countries, global developments and technological advancements for the HSR Network, motivation and novelties of this work. A brief literature review on the background of this work, including mathematical tools such as picture fuzzy numbers and MCDM methods, is discussed in Section 2. Section 3 describes the preliminary ideology on Picture fuzzy numbers. The decision-making technique, DEMATEL, is covered in Section 4. The selection of criteria for the adoption of the high-speed rail network is fully discussed in Section 5. Section 6 and Section 7 mention the model formulation, data collection and numerical illustration, respectively, in this research work. Then, the implications of the research are indicated in Section 8. And finally, the conclusions and future research scope are perfectly explained in Section 9.

2. Literature survey of on this study

This portion mainly describes the literature review for this study. First discussed different studies on the High-Speed Rail, then studies on the background of fuzzy set and especially its extension, namely Picture Fuzzy Numbers (PFNs). Lastly, we surveyed MCDM methodology and the MCDM method used was the DEMATEL methodology.

2.1 Background on the High-Speed Rail

High-speed rail [6] is a state-of-the-art transportation system for rapid passenger travel. It runs mainly at a speed of 250 km per hour (155 kmph). This system significantly reduces travel times, mainly between cities. Therefore, it has developed as a competitive alternative to air and road transport. It ensures the comfort and safety of passengers. High-speed trains have such an impact on road transport that they help reduce road congestion and also minimise carbon emissions. It is almost impossible to launch this system without an enduring development [7]. China, France and Japan are among the countries [8] that have pioneered significant HSR networks. Seeing them, this network service was later launched in various developing nations. This results in improved end-to-end capabilities between regions, which helps to stimulate economic growth. It also helps to meet increasing mobility and environmental demands as technology advances. Table 1 represents the studies on the high-speed rail network.

Table 1
Some studies on the high-speed rail network

Author	Year	Methodology	Application Area
[9] Cismaru, D. C. et al.	2008	Structural diagram with SIMULINK model	Useful movement of the high-speed trains
[7] Liu, Y. et al.	2019	Geometric law of the line shape of automobile body	Design geometry in high-speed rail design
[10] Yu, X et al.	2021	High-speed rail freight train organization plan	High-speed railways for the needs of various transport products
[8] Xu, L. et al.	2022	The structural characteristics of HSR network in China	Development of China's high-speed rail network based on data from 2008 to 2020
[6] Zhou, W. et al.	2022	A line-planning optimization	Merging the newly built railway lines for high-speed railway networks
[11] Zhen, L. et al.	2023	A metaheuristic algorithm	Network planning and freight allocation barriers for high-speed railway express systems
[12] Yuan, F. et al.	2025	GAN network based on the attention mechanism	Integrating the concept of service quality of high-speed railways with the SERVQUAL model
[13] Chen, Z. et al.	2025	Two-Level Spatiotemporal Network	Improving the timetables resilience of high-speed railway

2.2 *Background on Picture fuzzy numbers*

Fuzzy numbers [14] are used to get an accurate representation of work in uncertain environments of the decision-making process. More specifically, we use Picture Fuzzy Numbers (PFNs) here. This is a recently developed tool to handle ambiguity, which is a direct extension of Intuitionistic Fuzzy Set (IFS). This makes it possible to make successful decisions for various reasons and in various situations, even in an environment of ambiguity and uncertainty. There are several applications for fuzzy sets in numerous fields, including series solutions, differential equations, and integral equation solving, among others. A brief literature review on Picture Fuzzy Numbers (PFNs) is detailed in Table 2.

Table 2
Literature review on Picture Fuzzy Numbers (PFNs)

Author	Year	Application Area
[15] Garg, H.	2017	Application in multicriteria decision-making with some picture fuzzy aggregation operators
[16] Wang, C. et al.	2017	Application in multiple attribute decision making with some geometric aggregation operators in uncertain environment
[17] Wei, G.	2018	Application to multiple attribute decision making with picture fuzzy hamacher aggregation operators
[18] Si. A., et al.	2019	Application of sorting the Picture Fuzzy Numbers in decision-making issues
[19] Dinghong, P. et al.	2019	Application in renewable energy selection process
[20] Jana, C. et al.	2019	Application to madm methodology
[21] Peng, X. et al.	2021	Application on China's stock market bubble warning
[22] Rong, Y. et al.	2022	Evaluation of emergency management scheme with multiple attribute decision-making process in uncertain environment
[23] Sandra, M. et al.	2025	Application on an intelligent framework for sustainable C and D waste management system

2.3 *Background on DEMATEL methodology*

Multi-Criteria Decision Making (MCDM) is a methodology by which the ideal option is selected from various alternatives by properly considering various criteria. It can sometimes be contradictory or unclear. This method is mainly effective for dealing with complex situations. It is mainly used in situations where the solutions can be both quantitative and qualitative. The popular and most widely used MCDM methods in this decision-making process are Decision making trial and evaluation laboratory (DEMATEL), Analytic Hierarchy Process (AHP), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), Analytic Network Process (ANP), Complex Proportional Assessment (COPRAS), Entropy, Criteria Importance Through Intercriteria Correlation (CRITIC), Combined Compromise Solution (CoCoSo), Weighted Aggregates Sum Product Assessment (WASPAS), Multi-Objective Optimization Ratio Analysis (MOORA), VIekriterijumsko KOmpromisno Rangiranje (VIKOR), Stepwise Weight Assessment Ratio Analysis (SWARA) and others. Now, we give a brief summary of some potential research work for the MCDM method in various fuzzy environments below Table 3.

Table 3
Literature Review on various MCDM methods with their applications

Authors	**Year**	**Uncertainty**	**MCDM methods**	**Application Area**
[24] Tsai, W. H. et al.	2010	Crisp number	DEMATEL & ANP	Application for selecting cost of quality model growth
[25] Morote, A. N. et al.	2011	Trapezoidal fuzzy number (TrFN)	AHP	Application to combined cooling, heating and power production equipments
[26] Karabasevic, D. et al.	2015	Crisp number	SWARA & MULTIMOORA	Candidate selection in the mining domain
[27] Hafezalkotob, A. et al.	2016	Crisp number	MULTIMOORA & Shannon entropy	Application on material selection
[28] Wei, G.et al.	2020	Probabilistic uncertain linguistic term sets (PULTS)	GRA & CRITIC	Application for electric vehicle charging station's ideal site planning
[29] Ghorui, N. et al.	2021	Hesitant fuzzy set	AHP & TOPSIS	Application on the risk factor testing of the spread of COVID-19
[30] Wang, P. et al.	2022	TrFN	TOPSIS-CRITIC & ENTROPY	A new hybrid evaluation framework is suggested
[31] Alzahrani, F. A. et al.	2023	Neutrosophic number & TRNN	AHP, TOPSIS & COPRAS	Proper location Selection for Women University
[32] Gazi, K. et al.	2024	Spherical fuzzy number	Entropy & VIKOR	Hospital location selection in Saudi Arabia
[33] Genç, C. et al.	2024	No	AHP, MOORA and MAUT	Distinguish the business in the hotel industry using MCDM methodologies
[34] Çalikoğlu, C. et al.	2024	Bilinear ordering	TOPSIS	Investigate the Human Development Index (HDI) and Sustainable Development (SD) in countries in the European Union

The DEMATEL methodology is a weighted multicriteria decision-making (MCDM) process. It helps evaluate and visualize the relationships between criteria within a system. Its goal is to recognise cause-and-effect relationships, especially when interacting with intertwined, complex problems. Here, we provide a brief summary of some research papers on DEMATEL methodology in various fuzzy environments, as presented in Table 4 below.

Table 4
Some recent studies on the DEMATEL method with respective applications

Authors	Year	Uncertainty	MCDM methods	Application Area
[35] Shieh, J. I. et al.	2010	Crisp number	DEMATEL	Determining the key success factors of the quality of hospital service
[36] Tsai, W. H. et al.	2010	Crisp number	DEMATEL & ANP	Application on selecting cost of quality model development
[37] Samvedi, A. et al.	2013	Fuzzy set theory	DEMATEL	Application on supply chain risk management
[38] Tsai, W. H. et al.	2013	Crisp number	ANP & DEMATEL	Explore Risk Control and Information Technology Auditing in an Enterprise Resource Planning Context
[39] Nithyanandham, D. et al.	2021	Bipolar fuzzy graph	DEMATEL	Structuring DEMATEL process under bipolar uncertain environment
[40] Kashyap, A. et al.	2022	Crisp number	DEMATEL	Identifying the obstacles to the execution of circularity in the aluminium industry
[41] Bongo, M. F. et al.	2023	Crisp number	BWM & DEMATEL	Assessing the Performance-shaping factors of air traffic controllers
[42] Gazi, K. H. et al.	2025	Pentagonal fuzzy number (PFN)	DEMATEL	Compute the effective factor for women in sport sector
[43]Saraswathi Appasamy	2026	Triangular fuzzy number	DEMATEL	An Analysis of the DEMATEL process in ambiguity environment

3. Preliminary Ideology on Picture fuzzy number

In this section, we will review important basic concepts like fuzzy sets, fuzzy numbers, extensions of fuzzy sets, i.e., intuitionistic fuzzy sets and neutrosophic sets. Additionally, we will describe picture fuzzy sets and picture fuzzy numbers, exploring their definitions and operations with appropriate examples.

3.1 *Fuzzy sets and fuzzy numbers*

Whereas fuzzy numbers are used for quantification, fuzzy sets are applied for classification. The initial concept of fuzzy sets was first encountered by Lotfi A. Zadeh [14] in 1965. He proposed it as an extension of the classical idea of sets. Fuzzy sets and fuzzy numbers are used in various fields in real life, such as decision making [44], industrial engineering [45], control Systems, healthcare, skill Assessment, etc.

Definition 1. ***Fuzzy set*** *[46]*

Assume a collection of items, generally called $\mathscr{X}$. Let's now consider a fuzzy set, $\tilde{\mathscr{Q}}$, in this pairs $\mathscr{X}$ can be shown using bunch that consist of these,

$$\tilde{\mathscr{Q}} = \{(z, \mu_{\tilde{\mathscr{Q}}}(z)) : z \in \mathscr{X}\} \tag{1}$$

where, $\mu_{\tilde{\mathscr{Q}}}(z)$ is known as the membership function for the fuzzy set $\tilde{\mathscr{Q}}$. This function helps us to give each item in the set $\mathscr{X}$ a rate between the scale of 0 to 1. This rate indicates how much each member belongs to the given fuzzy set.

Definition 2. ***Fuzzy number*** *[46]*

A fuzzy number [42] is actually the generalization of a regular real number. Let us choose a fuzzy set, $\tilde{\mathscr{Q}}$, defined within the set of real numbers $\mathbb{R}$, which is also a universal set. If $\tilde{\mathscr{Q}}$ is classified as a fuzzy number, there exists an element $z \in \tilde{\mathscr{Q}}$, and it meets the requirements listed below,

- *(i) The membership function $\mu_{\tilde{\mathscr{Q}}}(z)$ of $\tilde{\mathscr{Q}}$ emphasises continuity piece by bit.*
- *(ii) $\tilde{\mathscr{Q}}$ is limited or restricted within a certain scale, which exhibits the support $\mu_{\tilde{\mathscr{Q}}}(z) > 0$.*
- *(iii) $\tilde{\mathscr{Q}}$ qualifies convexity within its fuzzy set structure.*
- *(iv) $\tilde{\mathscr{Q}}$ needs to have a normal fuzzy set and its height is $\mu_{\tilde{\mathscr{Q}}}(z) = 1$.*

Example 1. *In everyday life, we use words like "tall person", "warm weather", "good boy/ girl", etc. But the words "tall", "warm" and "good" can differ from person to person. This is where the concept of fuzzy logic comes from. Here are four sets of summer days, meaning "the warm days" being considered, which are $\mathscr{Y} = \{34°C, 36°C, 38°C, 40°C\}$. We can define the fuzzy set $\tilde{\mathscr{G}}$ of "the warm days" with the membership function, i.e., $\mathscr{G} = \{(34, 0.1); (36, 0.4); (38, 0.8); (40, 1.0)\}$. From the above range, it is clear that the first day with a temperature $34°C$ is considered "less warm day" with degree 0.1 and the fourth day with $40°C$ is "fully/extremely warm day" with degree 1.*

3.2 Extension of fuzzy set

Intuitionistic fuzzy sets and neutrosophic fuzzy sets are extensions of fuzzy set theory that better manage uncertainty. More specifically, it is said that a neutrosophic fuzzy set is a generalization of both fuzzy sets and intuitionistic fuzzy sets. In 1986, Krassimir T. Atanassov [47] introduced Intuitionistic Fuzzy Set (IFS) and then the neutrosophic fuzzy set was invented by Florentin Smarandache [48] in 1998.

Definition 3. ***Intuitionistic fuzzy set (IFS)*** *[47]*

Consider $\mathscr{X}$ to be a universal set. Then, there exist an element $z_i \in \mathscr{X}$ of an Intuitionistic Fuzzy Set (IFS) $\tilde{\mathcal{I}}$ on $\mathscr{X}$ is defined as,

$$\tilde{\mathcal{I}} = \{(z_i, \mu_{\tilde{\mathcal{I}}}(z_i), \nu_{\tilde{\mathcal{I}}}(z_i)) : z_i \in \mathscr{X}\}, \tag{2}$$

where the membership function and nonmembership function are referred to as $\mu_{\tilde{\mathcal{I}}}(z_i) : \mathscr{X} \to [0, 1]$ and $\nu_{\tilde{\mathcal{I}}}(z_i) : \mathscr{X} \to [0, 1]$, respectively, of $\tilde{\mathcal{I}}$ in the universal set $\mathscr{X}$. It follows the conditions, $0 \leq \mu_{\tilde{\mathcal{I}}}(z_i), \nu_{\tilde{\mathcal{I}}}(z_i) \leq 1$ and $0 \leq \mu_{\tilde{\mathcal{I}}}(z_i) + \nu_{\tilde{\mathcal{I}}}(z_i) \leq 1$.

Remark 1. *The degree of hesitation or intuitionistic fuzzy index can be indicated from the Intuitionistic fuzzy set. If this can be denoted by $\pi_{\tilde{\mathcal{I}}}(z_i)$, then the equation (2) gives that, $\pi_{\tilde{\mathcal{I}}}(z_i) = 1 - \mu_{\tilde{\mathcal{I}}}(z_i) - \nu_{\tilde{\mathcal{I}}}(z_i)$ with the condition $\pi_{\tilde{\mathcal{I}}}(z_i) : \mathscr{X} \to [0, 1]$.*

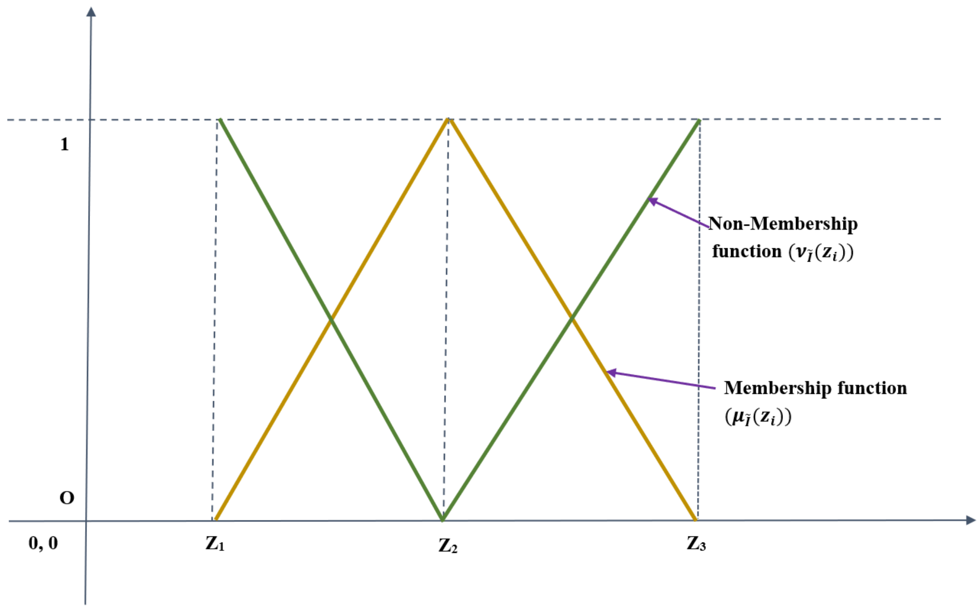

Fig. 1. Visual depiction of an Intuitionistic Fuzzy Set (IFS)

Definition 4. ***Neutrosophic set*** *[48]*

Let us choose $\mathscr{Z}$ to be a universal set. Then, a single-valued neutrosophic fuzzy set $\tilde{\mathcal{J}}$ on $\mathscr{Z}$ defined as,

$$\tilde{\mathcal{J}} = \{(z_j, \mu_{\tilde{\mathcal{J}}}(z_j), \omega_{\tilde{\mathcal{J}}}(z_j), \nu_{\tilde{\mathcal{J}}}(z_j)) : z_j \in \mathscr{Z}\}, \tag{3}$$

where, the degree of membership, inditerministic and nonmembership function follow the condition $\mu_{\tilde{\mathcal{J}}}(z_i), \omega_{\tilde{\mathcal{J}}}(z_i), \nu_{\tilde{\mathcal{J}}}(z_i) : \mathscr{Z} \to [0,1]$ of $\tilde{\mathcal{J}}$ in the universal set $\mathscr{Z}$. And, the primary condition of it is, $0 \leq \mu_{\tilde{\mathcal{J}}}(z_j) + \omega_{\tilde{\mathcal{J}}}(z_j) + \nu_{\tilde{\mathcal{J}}}(z_j) \leq 3$ for all $z_j \in \mathscr{Z}$.

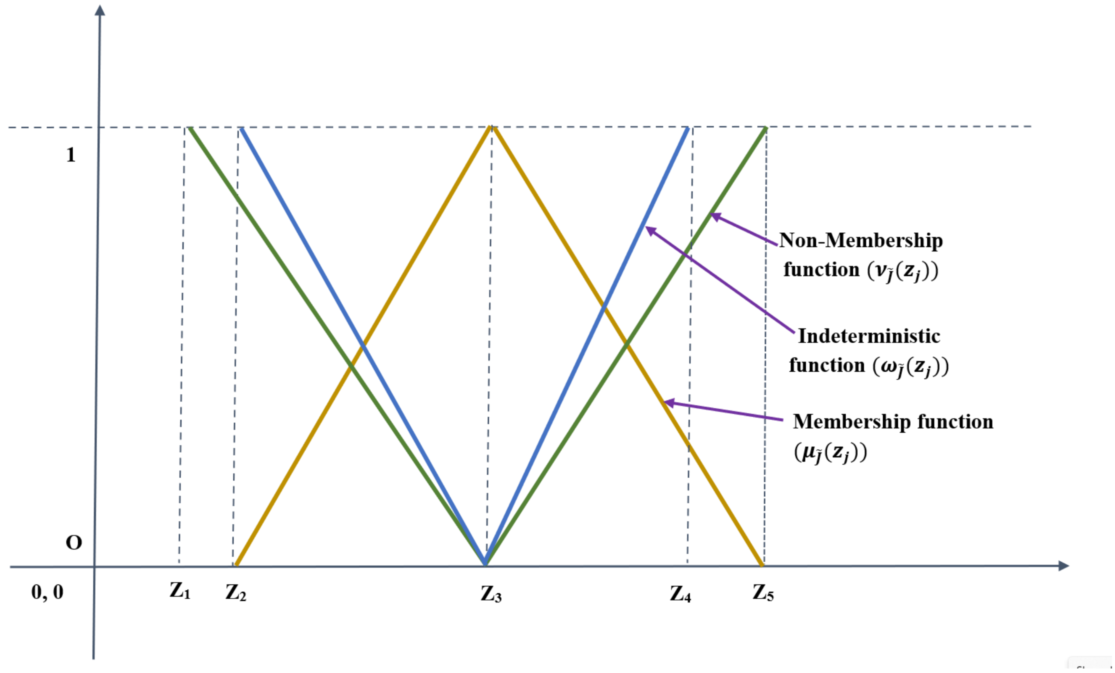

Fig. 2. Graphical depiction of a Neutrosophic Fuzzy Set (NFS)

3.3 *Picture fuzzy set and Picture fuzzy number*

A detailed discussion of the picture fuzzy set (PFS) [49] and the picture fuzzy number (PFN) [18] is presented here. PFS is an extension of a neutrosophic set where the summation of its membership functions is always less than or equal to $1\ (\leq 1)$ for all elements in the set. Picture fuzzy set (PFS) can be defined as follows:

Definition 5. ***Picture fuzzy set (PFS)*** *[15]*

Suppose that $\mathscr{Z}$ be a universal set. Therefore, a Picture fuzzy set (PFS) $\tilde{\mathcal{P}}$ on the universal set $\mathscr{Z}$ is of the form,

$$\tilde{\mathcal{P}} = \{(z_p, \mu_{\tilde{\mathcal{P}}}(z_p), \eta_{\tilde{\mathcal{P}}}(z_p), \nu_{\tilde{\mathcal{P}}}(z_p)) : z_p \in \mathscr{Z}\}, \tag{4}$$

then, there exists an element $z_p \in \mathscr{Z}$, where $\mu_{\tilde{\mathcal{P}}}(z_p), \eta_{\tilde{\mathcal{J}}}(z_p)$ and $\nu_{\tilde{\mathcal{P}}}(z_p)$ are the positive, neutral and negative membership function, respectively. Here, the basic satisfying conditions are, $\mu_{\tilde{\mathcal{P}}}(z_p), \eta_{\tilde{\mathcal{P}}}(z_p)$, $\nu_{\tilde{\mathcal{P}}}(z_p) : \mathscr{Z} \to [0,1]$ of $\tilde{\mathcal{P}}$ in $\mathscr{Z}$ and $0 \leq \mu_{\tilde{\mathcal{P}}}(z_p) + \eta_{\tilde{\mathcal{P}}}(z_p) + \nu_{\tilde{\mathcal{P}}}(z_p) \leq 1$.

Moreover, we get the degree of refusal membership function $(\kappa_{\tilde{\mathcal{P}}}(z_p))$ [18], from the picture fuzzy set $(\tilde{\mathcal{P}})$, as follows:

$$\kappa_{\tilde{\mathcal{P}}}(z_p) = 1 - \mu_{\tilde{\mathcal{P}}}(z_p) - \eta_{\tilde{\mathcal{P}}}(z_p) - \nu_{\tilde{\mathcal{P}}}(z_p) \tag{5}$$

where $z_p \in \mathscr{Z}$ be an arbitrary element and $\kappa_{\tilde{\mathcal{P}}}(z_p) \in [0,1], \forall z_p \in \mathscr{Z}$.

Definition 6. ***Picture fuzzy number (PFN)*** *[18]*

Let $\tilde{\mathcal{P}}$ be a picture fuzzy set (PFS) [15] on the set of real numbers $\mathbb{R}$, known as a picture fuzzy number (PFN) and it is denoted as,

$$\tilde{\mathcal{P}} = \{(z_p, \mu_{\tilde{\mathcal{P}}}(z_p), \eta_{\tilde{\mathcal{P}}}(z_p), \nu_{\tilde{\mathcal{P}}}(z_p)) : z_p \in \mathbb{R}\}. \tag{6}$$

And, $(\mu_{\tilde{\mathcal{P}}}(z_p), \eta_{\tilde{\mathcal{P}}}(z_p), \nu_{\tilde{\mathcal{P}}}(z_p), \kappa_{\tilde{\mathcal{P}}}(z_p))$ is also called the picture fuzzy number (PFN) for a fixed $z_p \in \tilde{\mathcal{P}}$ where $\mu_{\tilde{\mathcal{P}}}(z_p) + \eta_{\tilde{\mathcal{P}}}(z_p) + \nu_{\tilde{\mathcal{P}}}(z_p) + \kappa_{\tilde{\mathcal{P}}}(z_p) = 1$ and $\mu_{\tilde{\mathcal{P}}}(z_p), \eta_{\tilde{\mathcal{P}}}(z_p), \nu_{\tilde{\mathcal{P}}}(z_p), \kappa_{\tilde{\mathcal{P}}}(z_p) \in [0,1]$.

Remark 2. *Since the picture fuzzy is a numerical illustration of intuitionistic fuzzy, if we put $\eta_{\tilde{\mathcal{P}}}(z_p) = 0$ in Equation (4), then it will take the form of Equation (2) again. In a word, it is said that picture fuzzy set (PFS) reduces to intuitionistic fuzzy set (IFS).*

Remark 3. *Here, the above Equation (6) is a continuous picture fuzzy set as it deals with a domain, but in this paper, we apply a discrete picture fuzzy set (DPFS) that deals with individual elements.*

Example 2. *Suppose that, $\tilde{\mathscr{P}}$ be a PFN on the universal set $\mathscr{X} = \{6, 7, 8, 9\}$. Then, it is defined as, $\tilde{\mathscr{P}} = \{(6, 0.5, 0.3, 0.1), (7, 0.6, 0.2, 0.1), (8, 0.7, 0.1, 0.1), (9, 0.8, 0.1, 0.0)\}$. So, in the PFN $\tilde{\mathscr{P}}$, the component $6 \in \tilde{\mathscr{P}}$ has three membership values. The first one is positive membership function, i.e., $\mu_{\tilde{\mathscr{P}}}(6) = 0.5$, the neutral membership function is $\eta_{\tilde{\mathscr{P}}}(6) = 0.3$ and $\nu_{\tilde{\mathscr{P}}}(6) = 0.1$ be the negative membership function. This same condition will also apply for the elements $7, 8, 9 \in \tilde{\mathscr{P}}$. This is actually the accurate example for a discrete picture fuzzy set (DPFS).*

3.4 Reasons for choosing Picture Fuzzy Numbers (PFNs) over Neutrosophic Fuzzy Numbers (NFNs)

Assume that, $\tilde{\mathcal{J}} = \{(z_j, \mu_{\tilde{\mathcal{J}}}(z_j), \omega_{\tilde{\mathcal{J}}}(z_j), \nu_{\tilde{\mathcal{J}}}(z_j)) : z_j \in \mathscr{X}\}$ be a neutrosophic fuzzy set. We now describe some conditions that can be considered. These are

(i) If $0 \leq \mu_{\tilde{\mathcal{J}}}(z_j)+\omega_{\tilde{\mathcal{J}}}(z_j)+\nu_{\tilde{\mathcal{J}}}(z_j) \leq 1$, then the membership, inditerministic and nonmembership function are fully dependent.

(ii) If $0 \leq \mu_{\tilde{\mathcal{J}}}(z_j)+\omega_{\tilde{\mathcal{J}}}(z_j)+\nu_{\tilde{\mathcal{J}}}(z_j) \leq 3$, then the membership, inditerministic and nonmembership function are fully independent.

(iii) If $0 \leq \mu_{\tilde{\mathcal{J}}}(z_j)+\omega_{\tilde{\mathcal{J}}}(z_j)+\nu_{\tilde{\mathcal{J}}}(z_j) \leq 2$, then the membership, inditerministic and nonmembership function are partially independent. And, we can consider three cases from here: they are,

   a. If $0 \leq \mu_{\tilde{\mathcal{J}}}(z_j) + \omega_{\tilde{\mathcal{J}}}(z_j) \leq 1$ and $0 \leq \nu_{\tilde{\mathcal{J}}}(z_j) \leq 1$, then the membership, inditerministic functions are dependent. However, the nonmembership function is independent.

   b. If $0 \leq \mu_{\tilde{\mathcal{J}}}(z_j) + \nu_{\tilde{\mathcal{J}}}(z_j) \leq 1$ and $0 \leq \omega_{\tilde{\mathcal{J}}}(z_j) \leq 1$, then the membership, nonmembership functions are dependent. But the indeterministic function is independent.

   c. If $0 \leq \omega_{\tilde{\mathcal{J}}}(z_j)+\nu_{\tilde{\mathcal{J}}}(z_j) \leq 1$ and $0 \leq \mu_{\tilde{\mathcal{J}}}(z_j) \leq 1$, then the inditerministic, nonmembership functions are dependent. But the membership function is independent.

In this paper, we consider condition (i), in which the three functions are dependent. And, this is known as a Picture Fuzzy Number (PFN). In the PNSs, we also get the idea of the refusal membership function. It actually uses four membership degrees, which are more insightful in many real-life decision-making problems, particularly where abstention is common. Generally, these calculations are more straightforward due to the fixed constraint, making them more appropriate for beneficial uses like multi-criteria decision making (MCDM). For these reasons, we use PFNs in this study to solve this research problem.

3.5 Operations on PFSs and PFNs

In this section, we discussed the set operations of Picture Fuzzy Sets (PFSs) and the arithmetic operations of Picture Fuzzy Numbers (PFNs) in detail. The set operations on PFSs are as follows:

Definition 7. ***Set operations on PFSs***

Assume that $\mathscr{Z}$ is a universal set. Then, $\tilde{\mathscr{P}}_1 = \{(z_p, \mu_{\tilde{\mathscr{P}}_1}(z_p), \eta_{\tilde{\mathscr{P}}_1}(z_p), \nu_{\tilde{\mathscr{P}}_1}(z_p)) : z_p \in \mathscr{Z}\}$ and $\tilde{\mathscr{P}}_2 = \{(z_p, \mu_{\tilde{\mathscr{P}}_2}(z_p), \eta_{\tilde{\mathscr{P}}_2}(z_p), \nu_{\tilde{\mathscr{P}}_2}(z_p)) : z_p \in \mathscr{Z}\}$ be two PFSs on $\mathscr{Z}$. The operations of set $\tilde{\mathscr{P}}_1$ and $\tilde{\mathscr{P}}_2$ on $\mathscr{Z}$ are described below,

1. *Union of two PFSs:*

$$\begin{aligned}\tilde{\mathscr{P}}_1 \cup \tilde{\mathscr{P}}_2 = \big\{(z_p, \max\big\{\mu_{\tilde{\mathscr{P}}_1}(z_p), \mu_{\tilde{\mathscr{P}}_2}(z_p)\big\}, \min\big\{(\eta_{\tilde{\mathscr{P}}_1}(z_p), \eta_{\tilde{\mathscr{P}}_2}(z_p)\big\}, \\ \min\big\{\nu_{\tilde{\mathscr{P}}_1}(z_p), \nu_{\tilde{\mathscr{P}}_2}(z_p)\big\} : z_p \in \mathscr{Z}\big\}\end{aligned} \tag{7}$$

2. *Intersection of two PFSs:*

$$\begin{aligned}\tilde{\mathscr{P}}_1 \cap \tilde{\mathscr{P}}_2 = \big\{(z_p, \min\big\{\mu_{\tilde{\mathscr{P}}_1}(z_p), \mu_{\tilde{\mathscr{P}}_2}(z_p)\big\}, \min\big\{\eta_{\tilde{\mathscr{P}}_1}(z_p), \eta_{\tilde{\mathscr{P}}_2}(z_p)\big\}, \\ \max\big\{\nu_{\tilde{\mathscr{P}}_1}(z_p), \nu_{\tilde{\mathscr{P}}_2}(z_p)\big\} : z_p \in \mathscr{Z}\big\}\end{aligned} \tag{8}$$

3. *Complement of a PFSs:*

$$\begin{cases}\tilde{\mathscr{P}_1}^C = \left(\tilde{\mathscr{P}}_1\right)^C & = \left\{\left(z_p, \nu_{\tilde{\mathscr{P}}_1}(z_p), \eta_{\tilde{\mathscr{P}}_1}(z_p), \mu_{\tilde{\mathscr{P}}_1}(z_p)\right) : z_p \in \mathscr{Z}\right\} \\ \tilde{\mathscr{P}_2}^C = \left(\tilde{\mathscr{P}}_2\right)^C & = \left\{\left(z_p, \nu_{\tilde{\mathscr{P}}_2}(z_p), \eta_{\tilde{\mathscr{P}}_2}(z_p), \mu_{\tilde{\mathscr{P}}_2}(z_p)\right) : z_p \in \mathscr{Z}\right\}\end{cases} \tag{9}$$

Example 3. *Let $\tilde{\mathscr{R}}_1 = \{(z_x, 0.6, 0.1, 0.1) : z_x \in \mathscr{Z}\}$ and $\tilde{\mathscr{R}}_2 = \{(z_x, 0.5, 0.3, 0.1) : z_x \in \mathscr{Z}\}$ be two PFSs on the set of universe $\mathscr{Z}$, where $z_x \in \mathscr{Z}$. Then, the set operations on $\tilde{\mathscr{R}}_1$ and $\tilde{\mathscr{R}}_2$ are explained in below*

1. *The union of the two PFSs, $\tilde{\mathscr{R}}_1$ and $\tilde{\mathscr{R}}_2$ is, $\tilde{\mathscr{R}}_1 \cup \tilde{\mathscr{R}}_2 = \{(z_x, 0.6, 0.1, 0.1) : z_x \in \mathscr{Z}\}$.*
2. *The intersection of the two PFSs, $\tilde{\mathscr{R}}_1$ and $\tilde{\mathscr{R}}_2$ is, $\tilde{\mathscr{R}}_1 \cap \tilde{\mathscr{R}}_2 = \{(z_x, 0.5, 0.1, 0.1) : z_x \in \mathscr{Z}\}$.*
3. *The complement of the PFS, $\tilde{\mathscr{R}}_1$ is, $\tilde{\mathscr{R}_1}^C = \left(\tilde{\mathscr{R}}_1\right)^C = \{(z_x, 0.1, 0.1, 0.6) : z_x \in \mathscr{Z}\}$ and the complement of the PFS $\tilde{\mathscr{R}}_2$ is, $\tilde{\mathscr{R}_2}^C = \left(\tilde{\mathscr{R}}_2\right)^C = \{(z_x, 0.1, 0.3, 0.5) : z_x \in \mathscr{Z}\}$.*

Definition 8. ***Arithmetic operations on PFNs***

Let us choose $\mathscr{Z}$ to be the universal set of discourse. Consider, $\tilde{\mathscr{P}}_1 = \{(z_p, \mu_{\tilde{\mathscr{P}}_1}(z_p), \eta_{\tilde{\mathscr{P}}_1}(z_p), \nu_{\tilde{\mathscr{P}}_1}(z_p)) : z_p \in \mathscr{Z}\}$ and $\tilde{\mathscr{P}}_2 = \{(z_p, \mu_{\tilde{\mathscr{P}}_2}(z_p), \eta_{\tilde{\mathscr{P}}_2}(z_p), \nu_{\tilde{\mathscr{P}}_2}(z_p)) : z_p \in \mathscr{Z}\}$ be two PFSs on $\mathscr{Z}$. The arithmetic operations of two PFNs are, $\tilde{\mathscr{P}}_1$ and $\tilde{\mathscr{P}}_2$ on $\mathscr{Z}$ are explained as follows

1. *Addition of two PFNs:*

$$\begin{aligned}\tilde{\mathscr{P}}_1 \oplus \tilde{\mathscr{P}}_2 &= \left\{\left(z_p, \mu_{\tilde{\mathscr{P}}_1 \oplus \tilde{\mathscr{P}}_2}(z_p), \eta_{\tilde{\mathscr{P}}_1 \oplus \tilde{\mathscr{P}}_2}(z_p), \nu_{\tilde{\mathscr{P}}_1 \oplus \tilde{\mathscr{P}}_2}(z_p)\right) : z_p \in \mathscr{Z}\right\} \\ &= \left\{\left(z_p, \mu_{\tilde{\mathscr{P}}_1}(z_p), \eta_{\tilde{\mathscr{P}}_1}(z_p), \nu_{\tilde{\mathscr{P}}_1}(z_p)\right) : z_p \in \mathscr{Z}\right\} \\ &\qquad \oplus \left\{\left(z_p, \mu_{\tilde{\mathscr{P}}_2}(z_p), \eta_{\tilde{\mathscr{P}}_2}(z_p), \nu_{\tilde{\mathscr{P}}_2}(z_p)\right) : z_p \in \mathscr{Z}\right\} \\ &= \left\{\left(z_p, \mu_{\tilde{\mathscr{P}}_1}(z_p) + \mu_{\tilde{\mathscr{P}}_2}(z_p) - \mu_{\tilde{\mathscr{P}}_1}(z_p)\mu_{\tilde{\mathscr{P}}_2}(z_p),\right.\right. \\ &\qquad \left.\left.(1 - \mu_{\tilde{\mathscr{P}}_1}(z_p))\eta_{\tilde{\mathscr{P}}_2}(z_p) - \eta_{\tilde{\mathscr{P}}_1}(z_p)\eta_{\tilde{\mathscr{P}}_2}(z_p), \nu_{\tilde{\mathscr{P}}_1}(z_p)\nu_{\tilde{\mathscr{P}}_2}(z_p)\right) : z_p \in \mathscr{Z}\right\}\end{aligned} \tag{10}$$

2. *Scalar multiplication of PFN:*

$$
\begin{aligned}
\lambda\tilde{\mathscr{P}}_1 =& \lambda \times \tilde{\mathscr{P}}_1 = \left\{\left(z_p, \mu_{\lambda\tilde{\mathscr{P}}_1}(z_p), \eta_{\lambda\tilde{\mathscr{P}}_1}(z_p), \nu_{\lambda\tilde{\mathscr{P}}_1}(z_p)\right) : z_p \in \mathscr{Z}\right\} \\
=& \lambda \times \left\{\left(z_p, \mu_{\tilde{\mathscr{P}}_1}(z_p), \eta_{\tilde{\mathscr{P}}_1}(z_p), \nu_{\tilde{\mathscr{P}}_1}(z_p)\right) : z_p \in \mathscr{Z}\right\} \\
=& \left\{\left(z_p, (0.9 - \mu_{\tilde{\mathscr{P}}_1}(z_p))^\lambda, (1 - \eta_{\tilde{\mathscr{P}}_1}(z_p) - \nu_{\tilde{\mathscr{P}}_1}(z_p)), \nu_{\tilde{\mathscr{P}}_1}(z_p)^\lambda\right) : z_p \in \mathscr{Z}\right\}
\end{aligned} \tag{11}
$$

where $\lambda\,(> 0)$ *be a positive scalar number.*

3. *Multiplication of two PFNs:*

$$
\begin{aligned}
\tilde{\mathscr{P}}_1 \otimes \tilde{\mathscr{P}}_2 =& \left\{\left(z_p, \mu_{\tilde{\mathscr{P}}_1\otimes\tilde{\mathscr{P}}_2}(z_p), \eta_{\tilde{\mathscr{P}}_1\otimes\tilde{\mathscr{P}}_2}(z_p), \nu_{\tilde{\mathscr{P}}_1\otimes\tilde{\mathscr{P}}_2}(z_p)\right) : z_p \in \mathscr{Z}\right\} \\
=& \left\{\left(z_p, \mu_{\tilde{\mathscr{P}}_1}(z_p), \eta_{\tilde{\mathscr{P}}_1}(z_p), \nu_{\tilde{\mathscr{P}}_1}(z_p)\right) : z_p \in \mathscr{Z}\right\} \\
& \qquad \otimes \left\{\left(z_p, \mu_{\tilde{\mathscr{P}}_2}(z_p), \eta_{\tilde{\mathscr{P}}_2}(z_p), \nu_{\tilde{\mathscr{P}}_2}(z_p)\right) : z_p \in \mathscr{Z}\right\} \\
=& \left\{\left(z_p, \mu_{\tilde{\mathscr{P}}_1}(z_p)\mu_{\tilde{\mathscr{P}}_2}(z_p), (1 - \nu_{\tilde{\mathscr{P}}_1}(z_p))\eta_{\tilde{\mathscr{P}}_2}(z_p)\right.\right. \\
& \left.\left. + (1 - \nu_{\tilde{\mathscr{P}}_2})(z_p))\eta_{\tilde{\mathscr{P}}_1}(z_p) - \eta_{\tilde{\mathscr{P}}_1}\eta_{\tilde{\mathscr{P}}_2}, \nu_{\tilde{\mathscr{P}}_1} + \nu_{\tilde{\mathscr{P}}_2} - \nu_{\tilde{\mathscr{P}}_1}\nu_{\tilde{\mathscr{P}}_2}\right) : z_p \in \mathscr{Z}\right\}
\end{aligned} \tag{12}
$$

4. *Scalar power of PFN:*

$$
\begin{aligned}
\tilde{\mathscr{P}}_1{}^\lambda = (\tilde{\mathscr{P}}_1)^\lambda =& \left\{\left(z_p, \left(\mu_{\tilde{\mathscr{P}}_1}(z_p)\right)^\lambda, \left(\eta_{\tilde{\mathscr{P}}_1}(z_p)\right)^\lambda, \left(\nu_{\tilde{\mathscr{P}}_1}(z_p)\right)^\lambda\right) : z_p \in \mathscr{Z}\right\} \\
=& \left\{\left(z_p, \mu_{\tilde{\mathscr{P}}_1}(z_p), \eta_{\tilde{\mathscr{P}}_1}(z_p), \nu_{\tilde{\mathscr{P}}_1}(z_p)\right) : z_p \in \mathscr{Z}\right\}^\lambda \\
=& \left\{\left(z_p, (1 - \mu_{\tilde{\mathscr{P}}_1}(z_p))^\lambda, 1 - (1 - \eta_{\tilde{\mathscr{P}}_1}(z_p))^\lambda\right),\right. \\
& \left. 1 - \left(1 - \nu_{\tilde{\mathscr{P}}_1}(z_p))\right)^\lambda : z_p \in \mathscr{Z}\right\}
\end{aligned} \tag{13}
$$

where $\lambda\,(> 0)$ *be a positive scalar number.*

Example 4. *Consider that* $\mathscr{Z}$ *be the set of the universe. Then* $\tilde{\mathscr{R}}_1 = \{(z_x, 0.6, 0.1, 0.1) : z_x \in \mathscr{Z}\}$ *and* $\tilde{\mathscr{R}}_2 = \{(z_x, 0.5, 0.3, 0.1) : z_x \in \mathscr{Z}\}$ *be two PFSs on* $\mathscr{Z}$*, where* $z_x \in \mathscr{Z}$*. Then, the arithmetic operations on* $\tilde{\mathscr{R}}_1$ *and* $\tilde{\mathscr{R}}_2$ *are calculated as follows:*

1. *The addition of the two PFSs,* $\tilde{\mathscr{R}}_1$ *and* $\tilde{\mathscr{R}}_2$ *are,* $\tilde{\mathscr{R}}_1 \oplus \tilde{\mathscr{R}}_2 = \{(z_x, 0.8, 0.17, 0.01) : z_x \in \mathscr{Z}\}$.

2. *The scalar multiplication of the PFS,* $\tilde{\mathscr{R}}_1$ *is,* $\lambda\tilde{\mathscr{R}}_1 = \lambda \times \tilde{\mathscr{R}}_1 = \{(z_x, 0.09, 0.8, 0.01) : z_x \in \mathscr{Z}\}$, *where,* $2\,(> 0)$ *be a positive scalar number.*

3. *The multiplication of the two PFSs,* $\tilde{\mathscr{R}}_1$ *and* $\tilde{\mathscr{R}}_2$ *is,* $\tilde{\mathscr{R}}_1 \otimes \tilde{\mathscr{R}}_2 = \{(z_x, 0.06, 0.15, 0.19) : z_x \in \mathscr{Z}\}$.

4. *The scalar power of the PFS,* $\tilde{\mathscr{P}}_1{}^\lambda = \left(\tilde{\mathscr{P}}_1\right)^\lambda = \{(z_x, 0.16, 0.19, 0.19) : z_x \in \mathscr{Z}\}$, *where* $2\,(> 0)$ *be a positive scalar number.*

3.6 Score and accuracy functions on PFS

To measure the level of membership and ambiguity associated with an element in a fuzzy set, two functions are implemented: the score and the accuracy function. In this section, we will explain several score and accuracy functions previously used in various research papers. Also, we have proposed a novel score and accuracy function here, which is described in Equation (14) and Equation (15), respectively.

Definition 9. ***Proposed score function***

Let $\mathscr{Z}$ be the universal set. Consider, $\tilde{\mathscr{P}}_1 = \{(z_p, \mu_{\tilde{\mathscr{P}}_1}(z_p), \eta_{\tilde{\mathscr{P}}_1}(z_p), \nu_{\tilde{\mathscr{P}}_1}(z_p)) : z_p \in \mathscr{Z}\}$ be the PFS on the universal set $\mathscr{Z}$. Consequently, the suggested score function $\left(\mathscr{S}_{\tilde{\mathscr{P}}_1}\right)$ for PFS is defined as

$$\mathscr{S}_{\tilde{\mathscr{P}}_1}(z_p) = \mu_{\tilde{\mathscr{P}}_1}(z_p) - \frac{1}{2}\eta_{\tilde{\mathscr{P}}_1}(z_p) - \nu_{\tilde{\mathscr{P}}_1}(z_p) + \frac{1}{2}log\left(1 + \kappa_{\tilde{\mathscr{P}}_1}(z_p)\right) \tag{14}$$

where $0 \leq \mu_{\tilde{\mathcal{P}}}(z_p) + \eta_{\tilde{\mathcal{P}}}(z_p) + \nu_{\tilde{\mathcal{P}}}(z_p) \leq 1$ and $\kappa_{\tilde{\mathcal{P}}}(z_p)$ be the degree of refusal membership function.

Definition 10. ***Proposed accuracy function***

Let $\mathscr{Z}$ be the universal set. Consider, $\tilde{\mathscr{P}}_1 = \{(z_p, \mu_{\tilde{\mathscr{P}}_1}(z_p), \eta_{\tilde{\mathscr{P}}_1}(z_p), \nu_{\tilde{\mathscr{P}}_1}(z_p)) : z_p \in \mathscr{Z}\}$ be the PFS on $\mathscr{Z}$. Therefore, the proposed accuracy function $\left(\mathscr{A}_{\tilde{\mathscr{P}}_1}\right)$ for PFS is outlined as

$$\mathscr{A}_{\tilde{\mathscr{P}}_1}(z_p) = \mu_{\tilde{\mathscr{P}}_1}(z_p) + \frac{1}{2}\eta_{\tilde{\mathscr{P}}_1}(z_p) - \nu_{\tilde{\mathscr{P}}_1}(z_p) \tag{15}$$

where, $0 \leq \mu_{\tilde{\mathcal{P}}}(z_p) + \eta_{\tilde{\mathcal{P}}}(z_p) + \nu_{\tilde{\mathcal{P}}}(z_p) \leq 1$.

Now, for clear understanding and visualization, different scores and accuracy functions in different studies are reflected in Table 5.

Table 5

Score and accuracy functions on PFS in different studies

Author	**Score function** $\left(\mathscr{S}_{\tilde{\mathscr{P}}_1}(z_p)\right)$	**Accuracy function** $\left(\mathscr{A}_{\tilde{\mathscr{P}}_1}(z_p)\right)$
[18] Si. A., et al.	$(1 - \mu_{\tilde{\mathscr{P}}_1}(z_p)) - \nu_{\tilde{\mathscr{P}}_1}(z_p)$	No
[15]Garg, H.	$\mu_{\tilde{\mathscr{P}}_1}(z_p) - \eta_1(z_p) - \nu_{\tilde{\mathscr{P}}_1}(z_p)$	$\mu_{\tilde{\mathscr{P}}_1}(z_p) + \eta_1(z_p) + \nu_{\tilde{\mathscr{P}}_1}(z_p)$
[16] Wang, C. et al.	$\mu_{\tilde{\mathscr{P}}_1}(z_p) - \nu_{\tilde{\mathscr{P}}_1}(z_p)$	$\mu_{\tilde{\mathscr{P}}_1}(z_p) + \eta_1(z_p) + \nu_{\tilde{\mathscr{P}}_1}(z_p)$
[17] Wei, G.	$\frac{1}{2}\left(1 + \mu_{\tilde{\mathscr{P}}_1}(z_p) + \nu_{\tilde{\mathscr{P}}_1}(z_p)\right)$	No
[19] Dinghong, P. et al.	$\mu_{\tilde{\mathscr{P}}_1}(z_p) + \frac{1}{2}\eta_1(z_p) - \nu_{\tilde{\mathscr{P}}_1}(z_p)$	No
[21] Peng, X. et al.	$\mu_{\tilde{\mathscr{P}}_1}(z_p) - \nu_{\tilde{\mathscr{P}}_1}(z_p) + 1 + \frac{e^{\mu_{\tilde{\mathscr{P}}_1}(z_p) - \eta_{\tilde{\mathscr{P}}_1}(z_p) - \nu_{\tilde{\mathscr{P}}_1}(z_p)}}{1 + \kappa_{\tilde{\mathscr{P}}_1}(z_p)}$	No
[22] Rong, Y. et al.	$\frac{e^{\mu_{\tilde{\mathscr{P}}_1}(z_p) - \eta_{\tilde{\mathscr{P}}_1}(z_p) - \nu_{\tilde{\mathscr{P}}_1}(z_p)}}{2 - \mu_{\tilde{\mathscr{P}}_1}(z_p) - \eta_{\tilde{\mathscr{P}}_1}(z_p) - \nu_{\tilde{\mathscr{P}}_1}(z_p)}$	No
[20] Jana, C. et al.	$\frac{1}{2}\left(1 + \mu_{\tilde{\mathscr{P}}_1}(z_p) - \nu_{\tilde{\mathscr{P}}_1}(z_p)\right)$	$\mu_{\tilde{\mathscr{P}}_1}(z_p) - \nu_{\tilde{\mathscr{P}}_1}(z_p)$
This study	$\mu_{\tilde{\mathscr{P}}_1}(z_p) - \frac{1}{2}\eta_{\tilde{\mathscr{P}}_1}(z_p) - \nu_{\tilde{\mathscr{P}}_1}(z_p) + \frac{1}{2}log\left(1 + \kappa_{\tilde{\mathscr{P}}_1}(z_p)\right)$	$\mu_{\tilde{\mathscr{P}}_1}(z_p) + \frac{1}{2}\eta_{\tilde{\mathscr{P}}_1}(z_p) - \nu_{\tilde{\mathscr{P}}_1}(z_p)$

Remark 4. *The score function usually converts a fuzzy number into a crisp value. If the values of two score functions are the same, we can use the accuracy function to separate them.*

Example 5. *Assume that $\mathscr{Z}$ be the universal set. Then $\tilde{\mathscr{R}}_1 = \{(z_x, 0.6, 0.1, 0.1) : z_x \in \mathscr{Z}\}$ be the PFS on $\mathscr{Z}$ where $z_x \in \mathscr{Z}$. The proposed score function of $\left(\mathscr{S}_{\tilde{\mathscr{R}}_1}\right)$ in this paper is described below. From*

Equation (14) *we get*

$$\begin{aligned}\mathscr{S}_{\tilde{\mathscr{R}}_1}(z_x) =&0.6 - \frac{1}{2} \times 0.1 - 0.1 + \frac{1}{2}\log\left(1 + (1 - 0.6 - 0.1 - 0.1)\right)\\ =&0.6 - 0.05 - 0.1 + \frac{1}{2}\log(1.2)\\ =&0.6 - 0.05 - 0.1 + 0.04\\ =&0.49\end{aligned}$$

Now, the suggested accuracy function $\left(\mathscr{A}_{\tilde{\mathscr{R}}_1}\right)$ *for PFS from Equation* (15) *is defined as*

$$\begin{aligned}\mathscr{A}_{\tilde{\mathscr{R}}_1}(z_x) =&0.6 + \frac{1}{2} \times 0.1 - 0.1\\ =&0.6 + 0.05 - 0.1\\ =&0.6 + 0.05 - 0.1\\ =&0.55\end{aligned}$$

Here, we find a comparative Table 6 of the values of different scores and accuracy functions on PFS in different studies, which is available from Table 5 and shows with respect to $\tilde{\mathscr{R}}_1 = \{(z_x, 0.6, 0.1, 0.1) : z_x \in \mathscr{Z}\}$ on the universal set $\mathscr{Z}$ where $z_x \in \mathscr{Z}$.

Table 6
The comparison table of the value of score and accuracy functions on PFS in different studies from Table 5

Author	Score function $\left(\mathscr{S}_{\tilde{\mathscr{R}}_1}(z_x)\right)$	Accuracy function $\left(\mathscr{A}_{\tilde{\mathscr{R}}_1}(z_x)\right)$
[18] Si, A. et al.	0.3	No
[15] Garg, H.	0.4	0.8
[16] Wang, C. et al.	0.5	0.8
[17] Wei, G.	0.85	No
[19] Dinghong, P. et al.	0.55	No
[21] Peng, X. et al.	2.74	No
[22] Rong, Y. et al.	1.24	No
[20] Jana, C. et al.	0.75	0.5
This study	0.49	0.55

4. Fuzzy Decision Making Trial and Evaluation Laboratory (DEMATEL) Methodology

A collection of methodologies known as multi-criteria decision making (MCDM) is used to assess and choose the options based on a number of conflicting criteria. It assists decision-makers in prioritizing, ranking or selecting options when there is no single best alternative. There are some common decision making methods include AHP (Analytic Hierarchy Process) [31], TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) [31], PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) [50], DEMATEL (Decision Making Trial and Evaluation Laboratory) [36], etc. The process usually involves structuring the problem, assigning weights to criteria, and evaluating alternatives. Normalization and aggregation techniques are frequently employed to guarantee comparability among criteria. MCDM is widely applied in various business, engineering, environmental and policy-making decisions.

The Multi-Criteria Decision Making DEMATEL process was first invented by the Geneva Research Centre of the Battelle Memorial Institute [51]. Particularly, Fontela and Gabus [52] developed this method initially in the 1970s to analyse the complicated, intertwined problems. It is very useful in analysing the cause and effect relationships among the components of a model. This kind of method is known as structural modelling. In our study, we use Fuzzy DEMATEL to process the problem. This is actually a hybrid MCDM methodology merging fuzzy logic and DEMATEL to evaluate complex systems. The DEMATEL process is used across a wide range of fields in practice. This method is used for model development [36], supply chain risk management [37], air traffic controllers [41], e-learning programs [53], etc. Here, Figure 3 shows the clear visualization of the total DEMATEL approach.

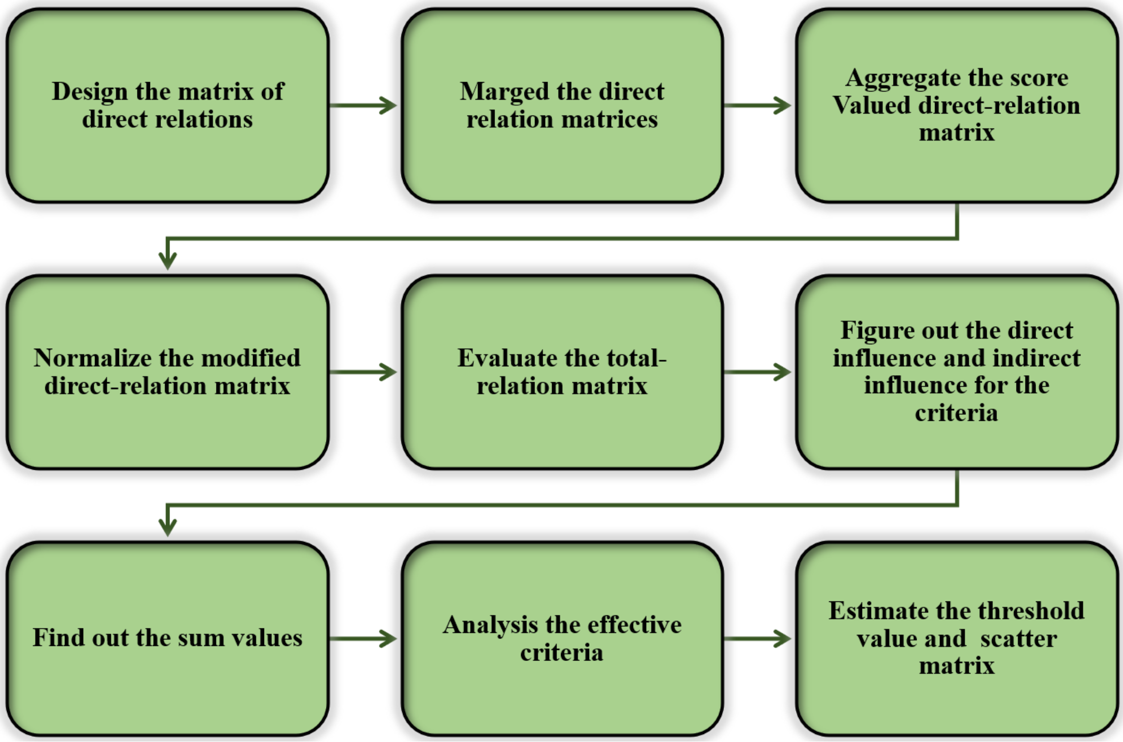

Fig. 3. Hierarchy structure of the DEMATEL process

Based on the knowledge and experience, there are t number of decision makers (DMs) who provide opinions in this research work. The following steps are in the DEMATEL method:

(i) Design the matrix of direct relations $(\mathscr{R}^d)$**:**
Direct relation matrix structured by δ-th DMs based on their knowledge and astute judgement in linguistic terms. The estimation of the interrelation between yth criteria with zth criteria is measured by the rate of comparison with the help of Table 7, where $y, z = 1, 2, \ldots, x$.There are t direct relation matrices $(\mathscr{R}^\delta)$ formulated in $x \times x$ order where $\delta = 1, 2, \ldots, t$. Here, the δ-th

direct relation matrix ($\mathscr{R}^\delta$) is displayed as,

$$\mathscr{R}^\delta = \begin{bmatrix} (\tilde{\Upsilon}_{11})^\delta & (\tilde{\Upsilon}_{12})^\delta & \dots & (\tilde{\Upsilon}_{1z})^\delta & \dots & (\tilde{\Upsilon}_{1x})^\delta \\ (\tilde{\Upsilon}_{21})^\delta & (\tilde{\Upsilon}_{22})^\delta & \dots & (\tilde{\Upsilon}_{2z})^\delta & \dots & (\tilde{\Upsilon}_{2x})^\delta \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ (\tilde{\Upsilon}_{y1})^\delta & (\tilde{\Upsilon}_{y2})^d & \dots & (\tilde{\Upsilon}_{yz})^\delta & \dots & (\tilde{\Upsilon}_{yx})^\delta \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ (\tilde{\Upsilon}_{x1})^\delta & (\tilde{\Upsilon}_{x2})^\delta & \dots & (\tilde{\Upsilon}_{xz})^\delta & \dots & (\tilde{\Upsilon}_{xx})^\delta \end{bmatrix}_{x\times x} \tag{16}$$

where $(\tilde{\Upsilon}_{yz})^\delta$ entry stands for the ratings of zth criteria with respect to yth criteria given by δ-th DM. Moreover, $(\tilde{\Upsilon}_{yz})^\delta$ be a picture fuzzy number that is presented in Equation 17, i.e.,

$$\begin{aligned} (\tilde{\Upsilon}_{yz})^\delta =& \{(z_p, \mu_{\tilde{\mathscr{P}}_1}(z_p), \eta_{\tilde{\mathscr{P}}_1}(z_p), \nu_{\tilde{\mathscr{P}}_1}(z_p))\}^\delta \\ =& \{((z_p)^\delta_{yz}, (\mu_{\tilde{\mathscr{P}}_1}(z_p))^\delta_{yz}, (\eta_{\tilde{\mathscr{P}}_1}(z_p))^\delta_{yz}, (\nu_{\tilde{\mathscr{P}}_1}(z_p)))^\delta_{yz}\} \\ =& \{(\mu_{\tilde{\mathscr{P}}_1}(z_p), \eta_{\tilde{\mathscr{P}}_1}(z_p), \nu_{\tilde{\mathscr{P}}_1}(z_p))\}^\delta_{yz} \end{aligned} \tag{17}$$

where, z_p be an arbitrary element and it belongs to the universal set $\mathscr{Z}$; $z_p \in \mathscr{Z}$ and $0 \leq \mu_{\tilde{\mathscr{P}}}(z_p) + \eta_{\tilde{\mathscr{P}}}(z_p) + \nu_{\tilde{\mathscr{P}}}(z_p) \leq 1$.

(ii) Marged the direct relation matrices ($\mathscr{R}_a$):
For future computation, all t number of direct matrices ($\mathscr{R}^\delta$) where $\delta = 1, 2, \dots, t$, pair up to form a single direct relation matrix ($\mathscr{R}_a$). After combining the every yzth entry $(_{yz})$, we establish single direct relation matrix ($\mathscr{R}_a$), that is

$$\begin{cases} (\mu_{\tilde{\mathscr{P}}_1}(z_p))_{yz} = & \min\limits_{\delta=1,2,\dots,t} \left\{(\mu_{\tilde{\mathscr{P}}_1}(z_p))^\delta_{yz}\right\} \\ (\eta_{\tilde{\mathscr{P}}_1}(z_p))_{yz} = & \max\limits_{\delta=1,2,\dots,t} \left\{(\eta_{\tilde{\mathscr{P}}_1}(z_p))^\delta_{yz}\right\} \\ (\nu_{\tilde{\mathscr{P}}_1}(z_p))_{yz} = & \min\limits_{\delta=1,2,\dots,t} \left\{(\nu_{\tilde{\mathscr{P}}_1}(z_p))^\delta_{yz}\right\} \end{cases} \tag{18}$$

and direct relation matrix ($\mathscr{R}_a$) looks like,

$$\mathscr{R}_a = \left[\tilde{\Upsilon}_{yz}\right]_{x\times x} = \begin{bmatrix} \tilde{\Upsilon}_{11} & \tilde{\Upsilon}_{12} & \dots & \tilde{\Upsilon}_{1z} & \dots & \tilde{\Upsilon}_{1x} \\ \tilde{\Upsilon}_{21} & \tilde{\Upsilon}_{22} & \dots & \tilde{\Upsilon}_{2z} & \dots & \tilde{\Upsilon}_{2x} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{\Upsilon}_{y1} & \tilde{\Upsilon}_{y2} & \dots & \tilde{\Upsilon}_{yz} & \dots & \tilde{\Upsilon}_{yx} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{\Upsilon}_{x1} & \tilde{\Upsilon}_{x2} & \dots & \tilde{\Upsilon}_{xz} & \dots & \tilde{\Upsilon}_{xx} \end{bmatrix}_{x\times x} \tag{19}$$

(iii) Aggregate the score valued direct-relation matrix ($\mathscr{R}_f$):
Find the de-fuzzified aggregated direct-relation matrix ($\mathscr{R}_f$) by de-fuzzified each entry of the aggregated direct-relation matrix ($\mathscr{R}_a$) with the help of the formula of the score function in Equation (14). So, the score valued direct-relation matrix ($\mathscr{R}_f$) is formulated as,

$$\mathscr{R}_f = [\Upsilon_{yz}]_{x\times x} \tag{20}$$

where Υ_{yz} is the score value of PFN $\tilde{\Upsilon}_{yz}$.

(iv) Normalize the modified direct-relation matrix $(\mathscr{R}^m)$:
Now, the modified direct-relation matrix $\mathscr{R}_f$ is normalized using equation (21). The normalized direct-relation matrix $(\mathscr{R}^m)$ is denoted as follows,

$$\mathscr{R}^m = \left[\Upsilon^m_{yz}\right]_{x\times x} = \left[\frac{\Upsilon_{yz}}{\max\limits_{1\le y\le x}\left\{\sum_{z=1}^{x}\Upsilon_{yz}\right\}}\right]_{x\times x} \tag{21}$$

where Υ^m_{yz} is the normalized value of Υ_{yz} and $y, z = 1, 2, \ldots, x$.

(v) Evaluate the total-relation matrix $(\mathscr{R}^l)$:
Calculate the total-relation matrix $(\mathscr{R}^l)$ from the normalized direct-relation matrix $(\mathscr{R}^m)$ is,

$$\begin{aligned}\mathscr{R}^l &= \lim_{\iota\to\infty}\left\langle \mathscr{R}^m + (\mathscr{R}^m)^2 + (\mathscr{R}^m)^3 + \cdots + (\mathscr{R}^m)^\iota\right\rangle\\ \text{or, } \mathscr{R}^l &= \mathscr{R}^m\left(\mathcal{I} - \mathscr{R}^m\right)^{-1} = \left[\beta_{yz}\right]_{x\times x}\end{aligned} \tag{22}$$

where $\mathcal{I}$ be the identity matrix of x-th order and β_{yz} is the yzth entry of the total-relation matrix $(\mathscr{R}^l)$.

(vi) Figure out the direct influence (Ω_y) and indirect influence (Ξ_z) for the criteria:
The sum of y-th row entries of the total-relation matrix $(\mathscr{R}^l)$ is defined as direct influence value (Ω_y), i.e.,

$$\Omega_y = \sum_{z=1}^{x}\beta_{yz} \tag{23}$$

where $y = 1, 2, \ldots, x$ and the sum of z-th column entries of the total-relation matrix $(\mathscr{R}^l)$ is indicated indirect influence value (Ξ_z), i.e.,

$$\Xi_z = \sum_{y=1}^{x}\beta_{yz} \tag{24}$$

where $z = 1, 2, \ldots, x$.

(vii) Find out the sum values:
Causal diagrams for every criterion are assessed from the data set. Each criterion's prominence value (Π_y) is found on the horizontal axis and is determined as follows,

$$\begin{aligned}\Pi_y &= \Omega_y + \Xi_z\\ &= \Omega_y + \Xi_y \text{ ; since, } y \text{ \& } z \text{ represent the number of criteria, then } y \simeq z\\ &= \sum_{z=1}^{x}\beta_{yz} + \sum_{z=1}^{x}\beta_{zy}\end{aligned} \tag{25}$$

and the relation value (Λ_y) of every criterion lies on the vertical axis and is determined as follows:

$$\begin{aligned}\Lambda_y &= \Omega_y - \Xi_z\\ &= \Omega_y - \Xi_y \text{ ; since, } y \text{ \& } z \text{ represent the number of criteria, then } y \simeq z\\ &= \sum_{z=1}^{x}\beta_{yz} - \sum_{z=1}^{x}\beta_{zy}\end{aligned} \tag{26}$$

where, Ω_y and Ξ_y are evaluated in previous steps where $y = 1, 2, \ldots, x$.

(viii) Analysis the effective criteria:
In this final phase, Π_y values highlighted the importance of the criteria, exhibiting the degree of relationship between each criterion and the others. Then, the criteria have a higher value of Π_y, indicating that they have a stronger relationship with other criteria. Concurrently, those criteria have a lower value of Π_y, showing that they have a weaker relationship with others.

On the other hand, the Λ_y value reveals the type of relation among the criteria. If the Λ_y value is positive $(+ve)$, then the criterion belongs to the dispatcher or cause group, that is, the yth criteria influence other criteria. And as a substitute, when the Λ_y value is negative $(-ve)$, the criteria belong to the receiver or effect group, that is, the yth criteria impacted by other criteria.

Finally, according to the Π_y and Λ_y values of the criteria, designed the causal diagram in XY plane, where $y = 1, 2, \ldots, x$. The causal diagram shows the direct and indirect effects of each criterion on the other criteria. Likewise, decision makers can set up the threshold value to determine the negligible effect of a criterion over all other criteria.

(ix) Estimate the threshold value (ι^t):
Now, the threshold value (ι^t) is computed from the total relation matrix $(\mathscr{R}^t)$ from Equation 22 with the help of the following formula, i.e.,

$$\iota^t = \frac{\sum_{y=1}^{x} \sum_{z=1}^{x} (\beta_{yz})}{x^2} \tag{27}$$

where $y, z = 1, 2, \ldots, x$.

(x) Evaluating scatter matrix $(\mathscr{S}^t)$:
From the total relation matrix $(\mathscr{R}^t)$ and threshold value (ι^t) with Equations (22) and (27), respectively, we find the Scatter matrix $(\mathscr{S}^t)$ is computed as follows,

$$yz^{th} \text{ entry} = \begin{cases} \surd \text{ (i.e., entry is relatable)} & ; \text{if } \iota^t \geq \beta_{yz} \\ \times \text{ (i.e., entry is not relatable)} & ; \text{if } \iota^t < \beta_{yz} \end{cases} \tag{28}$$

where $y, z = 1, 2, \ldots, x$. Therefore, the yz^{th} entry is relatable, implying that the y^{th} criterion is relatable with the z^{th} criterion; alternatively, it is not relatable. The scatter matrix $(\mathscr{S}^t)$ can be displayed by drawing the scatter matrix diagram.

5. Challenges for adopting High-Speed Rail Network

High-speed rail (HSR) [13] networks are essential in the current era. This is the super-fast and effective train system designed to travel at speeds normally above 250 km/h. Minimising pollution and traffic congestion provides a sustainable alternative to air and road travel. The HSR network [9] enhances economic growth by improving regional accessibility and connectivity.

There are several important considerations involved in selecting criteria for assessing the challenges of adopting high-speed railway networks in developing countries. It is essential to keep the surrounding environment in mind when selecting these. The construction and maintenance of this network are costly, making economic feasibility a key consideration. However, budget constraints and limited private-sector interest in developing countries should be considered, so the availability of funds for investment needs to be assessed. Advanced technology is a concern in developing countries because they rely heavily on foreign expertise in many areas. Such large projects require political stability and long-term commitment. The current existing railway situation, along with infrastructure preparation and land acquisition arrangements, plays an important role in this regard. Environmental impacts, including land use and emissions, should be assessed. Urban connectivity and existing transportation systems play a crucial role in the success of these larger projects. The selected criteria are presented in Figure 4 for a quick overview. These criteria for designing high-speed rail (HSR) networks [7] for developing countries, based on environmental conditions, are considered and described in detail below. They are,

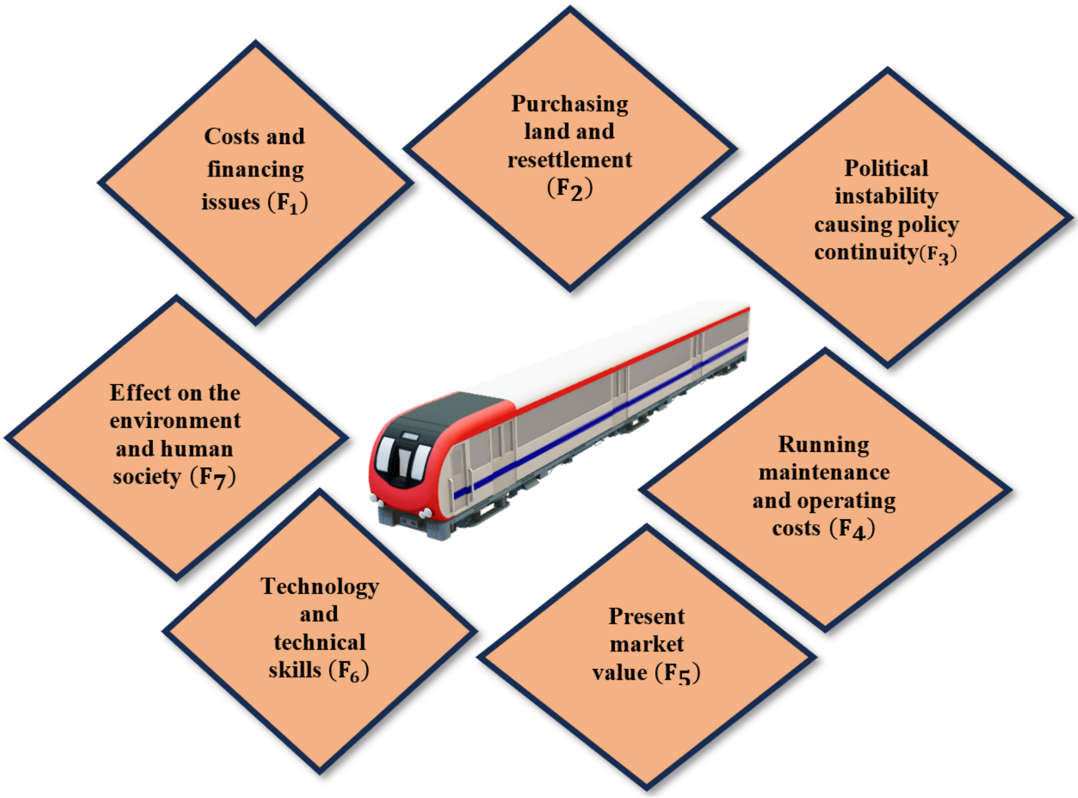

Fig. 4. The required criteria of the implied model

5.1 *High Capital Costs and Financing Issues* $(\mathscr{F}_1)$*:*

Building high-speed rail (HSR) networks [9] in developing nations is hindered by high capital costs and funding issues. Land acquisition is particularly necessary for the construction of HSR and infrastructure. Additionally, operating this system requires upfront investments, including specially designed trucks, advanced signalling systems, and rolling stock, totalling billions of dollars. In fact, investment costs, including infrastructure construction or maintenance, are very high, resulting in a long payback period and making it inconvenient. Its socio-economic impact is significant, including the reconfiguration of development sites, transportation infrastructure, changes in market share, and environmental pollution, among other factors. Prioritizing health and education makes it difficult to allocate adequate funds to developing countries since their budgets are extremely limited. This long fiscal period makes passenger transport uncertain, thereby affecting government investors. These economic challenges could slow down the implementation of this high-speed railway [8] or even lead to the project being cancelled or delayed.

5.2 *Purchasing Land and Resettlement* $(\mathscr{F}_2)$:

Developing countries face several challenges when adapting high-speed rail (HSR) networks [6]. Land purchasing and resettlement of affected communities play a key role. Land acquisition is fraught with legal disputes, unclear ownership, and high compensation claims. This increases costs and delays the construction of HSR. Furthermore, resettlement efforts for displaced populations have to address the social and economic impacts. These include adequate housing, restoration of livelihood and fair compensation. Limited institutional capacity and financial resources make this challenging. Moreover, political pressure, public resistance, and environmental concerns further complicate the process. This makes careful planning, transparent communication, and strong governance essential for successful HSR development [11].

5.3 *Political instability causing policy continuity* $(\mathscr{F}_3)$:

Political instability poses a significant challenge to the adaptation of high-speed rail (HSR) networks in developing countries [12]. It significantly disrupts policy continuity. Instability arises from frequent changes in government, inconsistent leadership and shifting political priorities. This can lead to the suspension, often delay or even cancellation of HSR schemes[9]. It is impossible to secure funding without long-term and stable commitments. The coordination of multi-year infrastructure projects becomes extremely complex as maintaining trust with investors is very difficult. It can also weaken the regulatory framework and disrupt land acquisition processes. As a result, public support for these projects decreases. This ultimately reduces the efficiency and success of HSR initiatives, hampering their effectiveness. Therefore, sustained political will and institutional resilience are crucial to overcoming these obstacles.

5.4 *Running maintenance and operating costs* $(\mathscr{F}_4)$:

Running maintenance and operation costs are crucial for developing countries to adapt high-speed rail (HSR) networks [10]. Proper operation of HSR systems ensures safety, reliability and high performance. These systems require continuous specialized maintenance, which requires advanced technology, skilled labour and sufficient financial resources. However, limited technical expertise creates this bottleneck. Furthermore, budget constraints, high cost of spare parts and other constraints hinder the development of HSR networks in developing countries [7]. In addition, passenger demand and revenue generation must be higher than expected. Otherwise, financial shortfalls may occur. Therefore, it can be said that capacity-building initiatives, sustainable funding models and efficient management are sufficient to address these long-term challenges.

5.5 *Present market value* $(\mathscr{F}_5)$:

The current market value of adopting high-speed rail (HSR) networks in developing countries [8] is significant. This is driven by increased mobility needs and increasing urbanization. HSR is considered a sustainable alternative to road and air transport. The implementation of this system reduces carbon emissions and travel times. The system stimulates economic growth by connecting major cities. This, in turn, can promote tourism, trade and also promote job creation. Investment in HSR infrastructure attracts foreign direct investment and promotes regional integration. Although the system has high initial costs, its long-term operational efficiency reduces transportation costs. Technical cooperation with developed countries increases local engineering capacity. HSR projects [6] also improve real estate values along the corridor. Government support and public-private partnerships strengthen the financial viability of operating the network. In general, HSR is a symbol of progress and modernity, consistent with the goals of sustainable development.

5.6 *Technology and technical skills* $(\mathscr{F}_6)$*:*

Technology and technical expertise are the primary obstacles to the adaption of high-speed rail (HSR) networks in developing countries [7]. HSR systems rely mainly on advanced technologies for signalling, operation, and safety. As a result, this often exceeds the current capacity of local industries and workers. High costs, intellectual property restrictions, and limited access to advanced technology pose barriers to the adoption of HSR networks. Furthermore, developing countries may face difficulties in accessing state-of-the-art technology due to over-reliance on foreign suppliers. In addition, there is a shortage of engineers, technicians and maintenance personnel specifically trained to operate these railways. This capacity shortage can lead to operational inefficiency, developed reliance on foreign expertise, and increased long-term costs. Building local technical capabilities through education, training programs and technology transfer is developed for sustainable HSR development [13].

5.7 *Effect on the environment and human society* $(\mathscr{F}_7)$*:*

The deployment of high-speed rail (HSR) networks [10] in developing countries has significant impacts on both the environment and human society. On the one hand, HSR systems [8] provide a clean alternative to road and air travel. This potentially reduces carbon emissions and eases traffic congestion. On the other hand, their construction can lead to habitat destruction, land degradation, and disruption of local ecosystems. Socially, HSR can boost economic growth by increasing regional connectivity. It displaces communities and changes traditional lifestyles. If its benefits are unevenly distributed, it can exacerbate inequalities. Careful environmental assessments, inclusive planning, and equitable resettlement policies become crucial to minimize negative impacts and maximize positive outcomes.

6. Model Formulation and Data Collection

The formulation of the model and the process of data collection for adapting the high-speed rail network in developing countries are described in this section.

6.1 *Model formulation for adapting high-speed rail network model*

The model of this paper is explained in this section. Here, we consider picture fuzzy numbers (PFNs) as a mathematical tool discussed in section 3.3 and the decision-making DEMATEL approach is indicated in Section 4, respectively. The specific discussion on criteria is mentioned in Section 5. There are seven key criteria for adapting a high-speed rail network in developing countries, according to the perspectives of proper literature review and decision experts. The direct relation matrices $(\mathscr{R}^d)$ with 7×7 order in linguistic terms given by the three DMs using Table 8. Figure 5 displays the pictorial flowchart of the proposed model. All three of those DMs possess professionalism, objectivity, and expertise in their own respective fields. They are,

DM1: A government officer.

DM2: An agent of technical agencies.

DM3: A financial stakeholder.

DM1 mainly sets budget constraints, environmental targets and social inclusion criteria, DM2 defines technical constraints, cost structure, etc. and DM3 mainly sets the budget of the model and helps to protect the public-private partnership. Based on their thinking, the model is properly considered and solved.

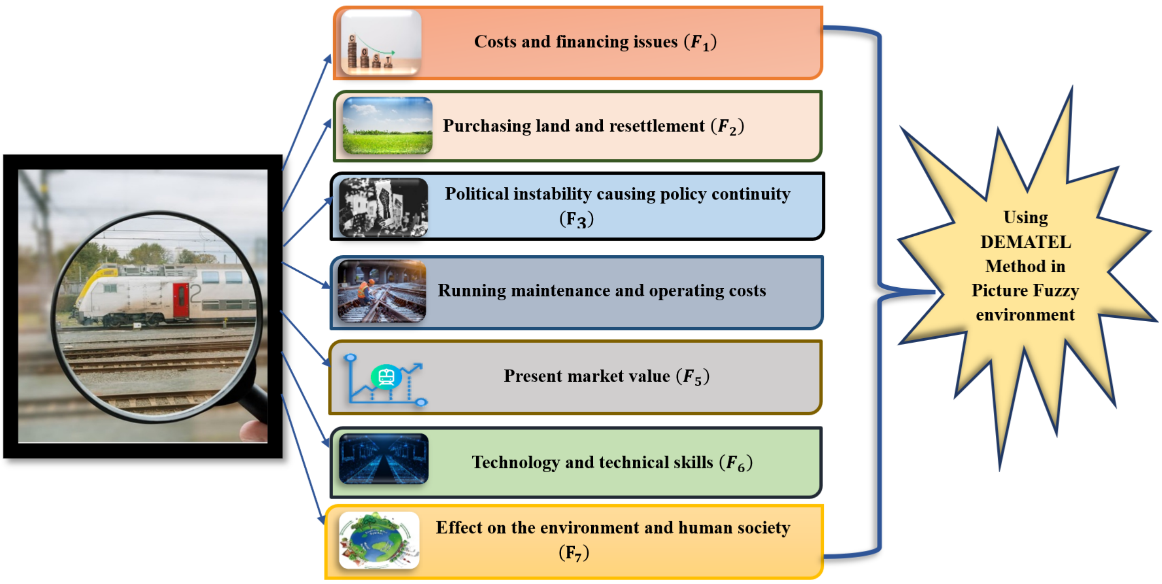

Fig. 5. The visual framework of the suggested model

6.2 *Data collection for this adaptation model*

Main data sources and data gathering methods used in this paper are analysed in this section.

Firstly, the data are collected by the three DMs in linguistic terms that we apply in the direct relation matrices $(\mathscr{R}^d)$ with the help of Table 7 and put them in the Table 8. Therefore, the direct relation matrices $(\mathscr{R}^d)$ were converted into Picture Fuzzy Numbers (PFNs) using Table 7 and developed for the DEMATEL methodology using Section 4.

Table 7
Conversion table between linguistic terms and Picture Fuzzy Numbers (PFNs)

Linguistic Terms	Picture Fuzzy Number (PFN)	Score value	Accuracy value
Severe Priority (SP)	(0.650, 0.150, 0.200)	0.375	0.525
Adequate Priority (AP)	(0.625, 0.125, 0.250)	0.313	0.438
High Priority (HP)	(0.600, 0.100, 0.300)	0.250	0.350
Regular Priority (RP)	(0.575, 0.075, 0.350)	0.188	0.263
Less Priority (LP)	(0.550, 0.050, 0.400)	0.125	0.175
Inadequate Priority (IP)	(0.525, 0.025, 0.450)	0.063	0.088
Negligently Priority (NP)	(0.500, 0.000, 0.500)	0.000	0.000

Table 8
Matrix of direct relations ($\mathscr{R}^d$) by three DMs in the given linguistic terms

	Criteria vs Criteria	High capital costs and financing issues ($\mathscr{F}_1$)	Purchasing land and resettlement ($\mathscr{F}_2$)	Political instability causing policy continuity ($\mathscr{F}_3$)	Running maintenance and operating costs ($\mathscr{F}_4$)	Present market value ($\mathscr{F}_5$)	Technology and technical skills ($\mathscr{F}_6$)	Effect on the environment and human society ($\mathscr{F}_7$)
DM 1	High capital costs and financing issues ($\mathscr{F}_1$)	NP	AP	NP	AP	HP	HP	RP
	Purchasing land and resettlement ($\mathscr{F}_2$)	HP	NP	LP	AP	HP	IP	LP
	Political instability causing policy continuity ($\mathscr{F}_3$)	IP	HP	NP	LP	LP	IP	NP
	Running maintenance and operating costs ($\mathscr{F}_4$)	LP	SP	RP	NP	AP	HP	LP
	Present market value ($\mathscr{F}_5$)	SP	AP	IP	RP	NP	LP	NP
	Technology and technical skills ($\mathscr{F}_6$)	HP	NP	IP	HP	LP	NP	IP
	Effect on the environment and human society ($\mathscr{F}_7$)	RP	AP	IP	RP	HP	IP	NP

	Criteria vs Criteria	High capital costs and financing issues ($\mathscr{F}_1$)	Purchasing land and resettlement ($\mathscr{F}_2$)	Political instability causing policy continuity ($\mathscr{F}_3$)	Running maintenance and operating costs ($\mathscr{F}_4$)	Present market value ($\mathscr{F}_5$)	Technology and technical skills ($\mathscr{F}_6$)	Effect on the environment and human society ($\mathscr{F}_7$)
DM 2	High capital costs and financing issues ($\mathscr{F}_1$)	NP	HP	IP	HP	AP	SP	RP
	Purchasing land and resettlement ($\mathscr{F}_2$)	SP	NP	NP	HP	SP	LP	LP
	Political instability causing policy continuity ($\mathscr{F}_3$)	RP	SP	NP	IP	IP	LP	IP
	Running maintenance and operating costs ($\mathscr{F}_4$)	IP	AP	RP	NP	HP	SP	RP
	Present market value ($\mathscr{F}_5$)	AP	SP	LP	LP	NP	NP	LP
	Technology and technical skills ($\mathscr{F}_6$)	AP	LP	LP	AP	RP	NP	NP
	Effect on the environment and human society ($\mathscr{F}_7$)	LP	SP	RP	RP	AP	LP	NP

	Criteria vs Criteria	High capital costs and financing issues ($\mathscr{F}_1$)	Purchasing land and resettlement ($\mathscr{F}_2$)	Political instability causing policy continuity ($\mathscr{F}_3$)	Running maintenance and operating costs ($\mathscr{F}_4$)	Present market value ($\mathscr{F}_5$)	Technology and technical skills ($\mathscr{F}_6$)	Effect on the environment and human society ($\mathscr{F}_7$)
DM 3	High capital costs and financing issues ($\mathscr{F}_1$)	NP	SP	RP	AP	HP	AP	LP
	Purchasing land and resettlement ($\mathscr{F}_2$)	AP	NP	LP	SP	HP	LP	IP
	Political instability causing policy continuity ($\mathscr{F}_3$)	LP	HP	NP	IP	IP	IP	LP
	Running maintenance and operating costs ($\mathscr{F}_4$)	IP	SP	NP	NP	HP	HP	IP
	Present market value ($\mathscr{F}_5$)	AP	SP	IP	LP	NP	IP	IP
	Technology and technical skills ($\mathscr{F}_6$)	AP	LP	IP	AP	LP	NP	RP
	Effect on the environment and human society ($\mathscr{F}_7$)	IP	SP	NP	LP	HP	IP	NP

7. Numerical Illustration

In this portion, the numerical illustration of the best effective criteria for adapting high-speed rail networks in developing countries is established and the results are evaluated here. We will discuss the numerical results and further analyze them. All mathematical calculations are carried out by using DEMATEL methodology, which is described in Section 5 with the optimization technique in Picture Fuzzy Numbers (PFNs) (see Section 3.3).

We will use the data that was collected from the three DMs in the form of a direct relation matrix $\mathscr{R}^d$ in Table 8. Then, we convert the data of the direct relation matrix $\mathscr{R}^d$ from linguistic terms to the picture fuzzy numbers (PFNs) with the help of Table 7. The aggregated direct relation matrix $(\mathscr{R}_a)$ is acquired by using Equation (19). After that, we find Equation (20) for calculating the score valued direct-relation matrix $\mathscr{R}_f$. Then, we apply the Equation (21) for building the normalized direct relation matrix $(\mathscr{R}^m)$, which is shown in Table 10. The total relation matrix $(\mathscr{R}^l)$ is computed in Table 11 by using Equation (22). Furthermore, we calculate the direct influence (Ω_y), indirect influence (Ξ_y), prominence value (Π_y) and relative value (Λ_y) by using Equation (24), Equation (25), Equation (26) and Equation (27), respectively and present the result in Table 12.

Table 9
Score valued aggregated direct-relation matrix $(\mathscr{R}_f)$

Criteria vs criteria	$\mathscr{F}_1$	$\mathscr{F}_2$	$\mathscr{F}_3$	$\mathscr{F}_4$	$\mathscr{F}_5$	$\mathscr{F}_6$	$\mathscr{F}_7$
Costs and financing issues $(\mathscr{F}_1)$	0.000	0.336	0.128	0.293	0.293	0.336	0.168
Purchasing land and resettlement $(\mathscr{F}_2)$	0.336	0.000	0.086	0.336	0.336	0.105	0.105
Political instability causing policy continuity $(\mathscr{F}_3)$	0.148	0.336	0.000	0.105	0.105	0.105	0.086
Running maintenance and operating costs $(\mathscr{F}_4)$	0.105	0.355	0.128	0.000	0.293	0.336	0.148
Present market value $(\mathscr{F}_5)$	0.355	0.355	0.105	0.168	0.000	0.086	0.086
Technology and technical skills $(\mathscr{F}_6)$	0.293	0.086	0.105	0.293	0.168	0.000	0.128
Effect on the environment and human society $(\mathscr{F}_7)$	0.148	0.355	0.128	0.168	0.293	0.105	0.000

Table 10
Normalizing the modified direct-relation matrix $(\mathscr{R}^m)$

Criteria vs criteria	$\mathscr{F}_1$	$\mathscr{F}_2$	$\mathscr{F}_3$	$\mathscr{F}_4$	$\mathscr{F}_5$	$\mathscr{F}_6$	$\mathscr{F}_7$
Costs and financing issues $(\mathscr{F}_1)$	0.000	0.184	0.070	0.161	0.161	0.184	0.092
Purchasing land and resettlement $(\mathscr{F}_2)$	0.184	0.000	0.047	0.184	0.184	0.058	0.058
Political instability causing policy continuity $(\mathscr{F}_3)$	0.081	0.184	0.000	0.058	0.058	0.058	0.047
Running maintenance and operating costs $(\mathscr{F}_4)$	0.058	0.195	0.070	0.000	0.161	0.184	0.081
Present market value $(\mathscr{F}_5)$	0.195	0.195	0.058	0.092	0.000	0.047	0.047
Technology and technical skills $(\mathscr{F}_6)$	0.161	0.047	0.058	0.161	0.092	0.000	0.070
Effect on the environment and human society $(\mathscr{F}_7)$	0.081	0.195	0.070	0.092	0.161	0.058	0.000

Table 11
Total-relation matrix $(\mathscr{R}^l)$

Criteria vs criteria	$\mathscr{F}_1$	$\mathscr{F}_2$	$\mathscr{F}_3$	$\mathscr{F}_4$	$\mathscr{F}_5$	$\mathscr{F}_6$	$\mathscr{F}_7$
Costs and financing issues $(\mathscr{F}_1)$	0.320	0.532	0.218	0.455	0.473	0.407	0.250
Purchasing land and resettlement $(\mathscr{F}_2)$	0.437	0.339	0.181	0.433	0.455	0.281	0.202
Political instability causing policy continuity $(\mathscr{F}_3)$	0.268	0.386	0.091	0.247	0.259	0.201	0.145
Running maintenance and operating costs $(\mathscr{F}_4)$	0.342	0.493	0.199	0.277	0.432	0.371	0.219
Present market value $(\mathscr{F}_5)$	0.422	0.470	0.176	0.338	0.270	0.247	0.179
Technology and technical skills $(\mathscr{F}_6)$	0.365	0.326	0.168	0.367	0.327	0.190	0.189
Effect on the environment and human society $(\mathscr{F}_7)$	0.332	0.471	0.186	0.332	0.408	0.245	0.130

Table 12
Sum values of the different criteria

Criteria	$\mathbf{\Omega_y} = \sum_{\mathbf{z=1}}^{\mathbf{x}} \beta_{\mathbf{yz}}$	$\mathbf{\Xi_y} = \sum_{\mathbf{y=1}}^{\mathbf{x}} \beta_{\mathbf{yz}}$	$\mathbf{\Pi_y} = \mathbf{\Omega_y} + \mathbf{\Xi_y}$	$\mathbf{\Lambda_y} = \mathbf{\Omega_y} - \mathbf{\Xi_y}$
Costs and financing issues $(\mathscr{F}_1)$	2.655	2.487	5.142	0.169
Purchasing land and resettlement $(\mathscr{F}_2)$	2.328	3.018	5.347	-0.690
Political instability causing policy continuity $(\mathscr{F}_3)$	1.597	1.219	2.816	0.378
Running maintenance and operating costs $(\mathscr{F}_4)$	2.333	2.448	4.781	-0.114
Present market value $(\mathscr{F}_5)$	2.102	2.625	4.727	-0.523
Technology and technical skills $(\mathscr{F}_6)$	1.933	1.941	3.874	-0.008
Effect on the environment and human society $(\mathscr{F}_7)$	2.103	1.315	3.418	0.789

Here, Table 12 displays the sum values of the considered criteria. The prominence value Π_y presents the significance of each criterion relative to others on the horizontal axis. We notice that purchasing land and resettlement $(\mathscr{F}_2)$ got the highest Prominence value. So, this is the most crucial criteria as it has the maximum relation with other criteria. Depending on this value we get that Costs and financing issues $(\mathscr{F}_1)$, Running maintenance and operating costs $(\mathscr{F}_4)$, Present market value $(\mathscr{F}_5)$, Technology and technical skills $(\mathscr{F}_6)$ and Effect on the environment and human society $(\mathscr{F}_7)$ has the second, third, fourth and fifth notable relation with respect to the other criteria respectively. Political instability, causing policy continuity $(\mathscr{F}_3)$, has the lowest relation with other criteria.

From Table 12, we also got the Relative value (Λ_y), which displayed the type of relation among the criteria. Here, the criteria Costs and financing issues $(\mathscr{F}_1)$, Political instability causing policy continuity $(\mathscr{F}_3)$ and Effect on the environment and human society $(\mathscr{F}_7)$ got positive $(+ve)$ Relative value. They belong to a cause or dispatcher group. On the other hand, we got negative $(-ve)$ relative value of the criteria Purchasing land and resettlement $(\mathscr{F}_2)$, Running maintenance and operating costs $(\mathscr{F}_4)$, Present market value $(\mathscr{F}_5)$ and Technology and technical skills $(\mathscr{F}_6)$, so, they belongs to the Effect group.

Figure 6 displays the causal diagram for the suggested model, which is used to determine the most effective criterion for adapting the high-speed rail network in developing countries.

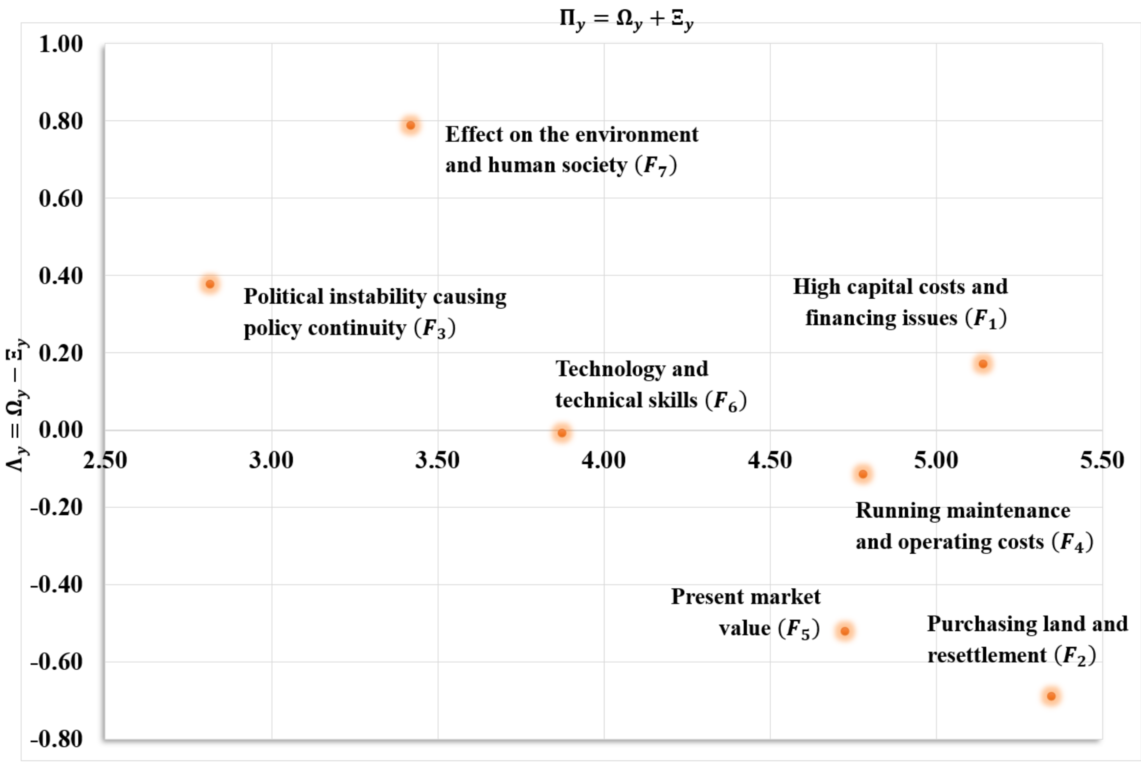

Fig. 6. Causal diagram created using each criterion's Π_y and Λ_y values

Here, we find the Threshold value (ι^t), using the Equation (27). So, the Threshold value (ι^t) is 0.30718.

With the help of the Total relation matrix $(\mathscr{R}^l)$ and Threshold value (ι^t), we get the Scatter matrix $(\mathscr{S}^t)$ which is shown in Table 13.

Table 13
Scatter matrix $(\mathscr{S}^t)$ of proposed DEMATEL process

Criteria	$\mathscr{F}_1$	$\mathscr{F}_2$	$\mathscr{F}_3$	$\mathscr{F}_4$	$\mathscr{F}_5$	$\mathscr{F}_6$	$\mathscr{F}_7$
Costs and financing issues $(\mathscr{F}_1)$	$\surd$	$\surd$	$\times$	$\surd$	$\surd$	$\surd$	$\times$
Purchasing land and resettlement $(\mathscr{F}_2)$	$\surd$	$\surd$	$\times$	$\surd$	$\surd$	$\times$	$\times$
Political instability causing policy continuity $(\mathscr{F}_3)$	$\times$	$\surd$	$\times$	$\times$	$\times$	$\times$	$\times$
Running maintenance and operating costs $(\mathscr{F}_4)$	$\surd$	$\surd$	$\times$	$\times$	$\surd$	$\surd$	$\times$
Present market value $(\mathscr{F}_5)$	$\surd$	$\surd$	$\times$	$\surd$	$\times$	$\times$	$\times$
Technology and technical skills $(\mathscr{F}_6)$	$\surd$	$\surd$	$\times$	$\surd$	$\surd$	$\times$	$\times$
Effect on the environment and human society $(\mathscr{F}_7)$	$\surd$	$\surd$	$\times$	$\surd$	$\surd$	$\times$	$\times$

Now, we draw the scatter diagram, which is shown in Figure 7 for proper visualization using the scatter matrix in Table 13. From this diagram, we have a clear understanding of the interrelation among all the criteria of the suggested model.

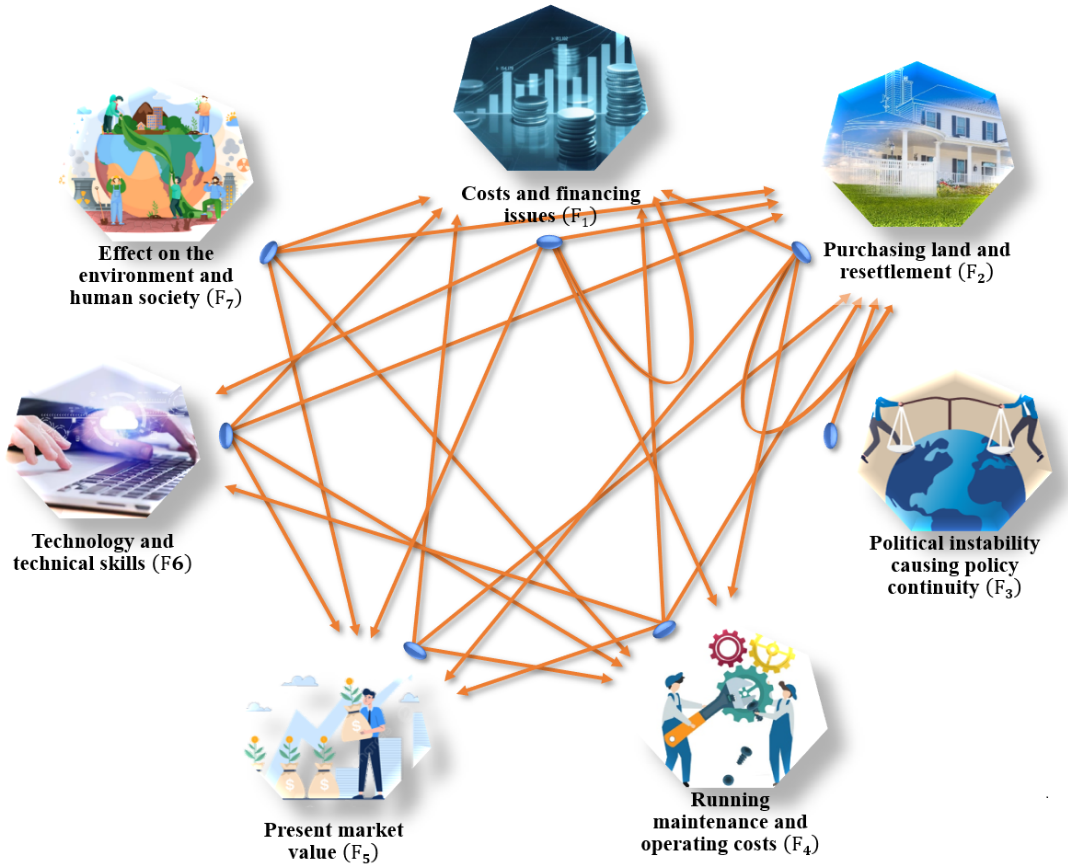

Fig. 7. Scatter matrix $(\mathscr{S}^t)$ diagram for the proposed HSR model

Remark 5. *From Table 13 and Figure 7, We can figure out that the criteria Costs and financing issues* $(\mathscr{F}_1)$ *is retable* $(\surd)$ *with* $(\mathscr{F}_1)$, $(\mathscr{F}_2)$, $(\mathscr{F}_4)$, $(\mathscr{F}_5)$ *and* $(\mathscr{F}_6)$, *Purchasing land and resettlement* $(\mathscr{F}_2)$ *is retable* $(\surd)$ *with* $(\mathscr{F}_1)$, $(\mathscr{F}_2)$, $(\mathscr{F}_4)$ *and* $(\mathscr{F}_5)$, *the criteria Political instability causing policy continuity* $(\mathscr{F}_3)$ *is retable* $(\surd)$ *with* $(\mathscr{F}_2)$, *the criteria Running maintenance and operating costs* $(\mathscr{F}_4)$ *is retable* $(\surd)$ *with* $(\mathscr{F}_1)$, $(\mathscr{F}_2)$, $(\mathscr{F}_5)$ *and* $(\mathscr{F}_6)$, *the criteria Present market value* $(\mathscr{F}_5)$ *is retable* $(\surd)$ *with* $(\mathscr{F}_1)$, $(\mathscr{F}_2)$ *and* $(\mathscr{F}_4)$, *the criteria Technology and technical skills* $(\mathscr{F}_6)$ *is retable* $(\surd)$ *with* $(\mathscr{F}_1)$, $(\mathscr{F}_2)$, $(\mathscr{F}_4)$ *and* $(\mathscr{F}_5)$ *and Effect on the environment and human society* $(\mathscr{F}_7)$ *is retable* $(\surd)$ *with* $(\mathscr{F}_1)$, $(\mathscr{F}_2)$, $(\mathscr{F}_4)$ *and* $(\mathscr{F}_5)$, *particularly.*

8. Research Implication

The research implications of a study usually refer to the application and significance of the results of the study. Here are various key research implications for adapting the high-speed rail (HSR) network in developing countries. This assessment requires research to analyze infrastructure development and cost-benefits. In fact, this research requires exploring cost-effective construction techniques that are suitable for local geography and materials. And in this case, social impact plays an important role. The role of these HSR networks on regional development, income growth or urbanization should also be assessed. Employment should be created for research. Social services and education should be analyzed as mobility changes. Among environmental considerations, the carbon footprint of HSR compared to other modes of transport can be investigated by studying the impact of rail construction on land systems, as well as on spatial and biodiversity. Training, participation in the local industry, and knowledge transfer are needed to research whether this imported network can be adequately adapted to the local context. To support its implementation, it is necessary to reform the legal and institutional regulatory system and study the role of public-private partnerships. Research is needed on passenger demand pricing models and integration with existing transportation scenarios to ensure connectivity and efficiency, while exploring issues such as foreign investment, government subsidies, etc. Analysis of this significance is necessary to develop a project literature resilience strategy while dealing with the biggest obstacles, such as cost increases and political instability. In addition, it is significant to specifically analyze social and cultural acceptance for developing an HSR network.

9. Conclusions and Future Research Scope

In this study, we find the most difficult challenges in adapting a high-speed rail network in developing countries. An MCDM methodology, namely DEMATEL, is applied to determine the factor weights. All required data are taken in linguistic terms from three wise decision makers' perspectives. All these data are gathered while remembering ambiguity in PFNs. Based on the requirements of seven different criteria, a model is set up. The core goal of this research paper is to establish a model for finding the best weighted criteria. Additionally, we proposed new score and accuracy functions for PFNs to determine the score-valued fuzzy function. After evaluating, it is found that purchasing land and resettlement $(\mathscr{F}_2)$ is the most weighted and Political instability causing policy continuity $(\mathscr{F}_3)$ is the least weighted criterion here. Moreover, the criteria Costs and financing issues $(\mathscr{F}_1)$, Political instability causing policy continuity $(\mathscr{F}_3)$ and Effect on the environment and human society $(\mathscr{F}_7)$ are in the cause group or dispatcher and the remaining criteria Purchasing land and resettlement $(\mathscr{F}_2)$, Running maintenance and operating costs $(\mathscr{F}_4)$, Present market value $(\mathscr{F}_5)$ and Technology and technical skills $(\mathscr{F}_6)$ are in effect group or receiver, respectively.

This research is significant for adopting high-speed railway networks in developing countries as it helps assess feasibility, route planning, and various economic effects. It guides the proper selection of technology suited to local conditions and modern infrastructure. Moreover, this study also indicates the environmental and social implications, making explicit sustainable development. It facilitates policy-making and investment decisions based on insights from required data. Without thorough this type of work, HSR projects risk inefficiency, public resistance and cost overruns.

9.1 Limitation of the work

This section primarily outlines the limitations of the present research. The main purpose of this study was to assess the best weighted criteria for adopting a high-speed rail network in developing countries. This work was conducted by three decision makers (DMs) and a small set of data sets for processing. Here, we consider PFNs to catch the uncertainty. Only seven criteria are considered and the alternatives are avoided for this work.

This DEMATEL methodology can evaluate the efficiency and dependability among the criteria using multiple conflicting criteria and decision-makers' datasets. Several weighted evaluation methodologies exist; however, the proposed model evaluated the relationships among the criteria in a scientific manner. Therefore, comparative analyses across different MCDM methods are challenging.

9.2 Future research scope

The future research outlines are covered elaborately in this section. In future work, this research could be extended in the following ways. The research was conducted only on adapting the challenges of high-speed rail networks in developing countries. Instead of Picture fuzzy numbers (PFNs), other fuzzy numbers may be considered as mathematical uncertain tools for capturing ambiguity. Multiple

MCDM approaches, such as AHP, Entropy, TOPSIS, MARCOS, MAIRCA, COPRAS, CoCoSo and SWARA, can be employed to compute the weights of the criteria and to rank all alternatives. In this paper, we have worked with the decisions of three decision makers, but in the future, more decision makers may be selected and their decisions may be prioritized for taking unbiased data sets. This study may be extended with multiple sub-criteria and alternatives based on a more thorough evaluation. Furthermore, in the future, we will conduct comparative analysis with the results of other known methods to compare the obtained results and then we will analyze the necessary comparative results. If we consider alternatives, we can also conduct functional and sensitivity analyses to assess their sustainability.

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Conflicts of Interest

The authors declare no conflicts of interest.

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