SCIENTIFIC OASIS

Decision Making Advances

Journal homepage: www.dma-journal.org
ISSN: 2956-2384

Confidence Level-Driven Dombi Aggregation Operators within the p,q-Quasirung Orthopair Fuzzy Environment for Sustainable Supplier Evaluation in Automotive Industry

Utpal Mandal[1], Mijanur Rahaman Seikh[2,*]

[1] Department of Mathematics & Statistics, Banasthali Vidyapith, Rajasthan-304 022, India
[2] Department of Mathematics, Kazi Nazrul University, Asansol 713340, India

ARTICLE INFO

Article history:
Received 19 January 2025
Received in revised form27 February 2025
Accepted 26 April 2025
Available online 8 May 2025

Keywords:
Multi-criteria group decision-making; p,q-QOFSs; Analytical hierarchy process; Confidence levels; Dombi aggregation operator; Sustainable supplier selection.

ABSTRACT

Selecting sustainable suppliers in the automotive industry is crucial for fostering environmental responsibility, cost efficiency, and ethical sourcing, all of which enhance long-term competitiveness and compliance with global sustainability standards. However, this process is a complex decision-making problem due to vague, uncertain, and imprecise data stemming from subjective expert judgments, incomplete information, dynamic market conditions, evolving regulations, and diverse stakeholder expectations. Traditional methods often fail to adequately capture these intricacies, necessitating more flexible and intelligent evaluation frameworks. To address this challenge, this study leverages p,q-quasirung orthopair fuzzy sets (p,q-QOFSs) to effectively model hesitation and ambiguity in expert assessments, while incorporating confidence levels to enhance reliability by accounting for varying decision-maker expertise. We propose confidence level-based Dombi weighted averaging (geometric) aggregation operators for p,q-QOFSs and develop a multi-criteria group decision-making model, with attribute weights determined using the Analytic Hierarchy Process (AHP). The model is validated through a case study in which three experts evaluate five automotive suppliers across eight sustainability criteria. A comparative analysis with existing methods demonstrates the superiority of the proposed approach, while sensitivity analysis confirms its robustness and stability under parameter variations.

*Corresponding author.
E-mail address: mrseikh@ymail.com

https://doi.org/10.31181/dma312025101

1. Introduction

The automotive industry stands as one of the largest and most influential sectors in the global economy. It plays a pivotal role in shaping economic growth, technological advancements, and employment generation. However, the industry is also a significant contributor to environmental concerns, including carbon emissions, excessive resource consumption, and industrial waste generation. With the rising emphasis on sustainability and corporate responsibility, the selection of sustainable suppliers has become an essential component of strategic decision-making within the automotive sector.

In recent years, stringent environmental regulations, ethical labor practices, and growing consumer awareness have compelled automotive manufacturers to adopt sustainable practices across their supply chains. The supplier selection process in this industry has evolved to include more than just the conventional factors of cost, quality, and delivery speed. A comprehensive and sustainable supplier selection approach should integrate environmental, social, and governance (ESG) factors to ensure responsible and ethical production. Sustainable supplier selection not only helps in minimizing the carbon footprint but also enhances resource optimization, cost efficiency, and corporate social responsibility initiatives.

As the demand for electric and hybrid vehicles continues to rise, there is an increasing emphasis on using environmentally friendly materials, adopting energy-efficient production methods, and ensuring sustainable waste management practices. Sustainable supplier selection should focus on procuring renewable and recyclable materials, ensuring compliance with green manufacturing standards, and fostering collaborations with suppliers that prioritize energy conservation and ethical labor practices. Moreover, suppliers should be assessed based on their ability to adopt circular economy principles, which promote recycling, reusing, and reducing waste throughout the production cycle.

To effectively achieve sustainability goals in supplier selection, the automotive sector needs to adopt Multi-Criteria Decision-Making (MCDM) techniques that assess suppliers using a clearly defined set of criteria. These criteria should include environmental sustainability metrics, social compliance indicators, and economic performance measures. However, given the intricate and multidimensional characteristics of the decision-making process, depending on just one decision-maker can result in partial judgments or less-than-optimal outcomes. Given the diverse factors involved in sustainable supplier selection, input from multiple experts across various fields is necessary to ensure a comprehensive and well-balanced evaluation. As a result, this problem falls under the domain of group decision-making, where collective expertise and consensus-driven methodologies are essential for achieving an optimal supplier selection strategy. Kolour, Momayezi and Momayezi [1] developed an integrated MCDM model to enhance supplier selection in public manufacturing. Wang *et al.* [2] developed an integrated DEMATEL-MARCOS model and applied it in sustainable food supplier selection. Mandal and Seikh [3] utilized the weighted distance-based approximation method to select supplier for a textile manufacturing company.

One of the key challenges in sustainable supplier selection for the automotive industry is the inherent uncertainty, vagueness, and incompleteness of real-world data. Many critical sustainability factors, such as environmental responsibility, ethical labor practices, and corporate social initiatives, are challenging to measure with exact numerical values. Additionally, expert assessments often rely on qualitative descriptions, such as “moderate durability” or “high environmental compliance,” which introduce subjectivity and ambiguity into the decision-making process. Furthermore, data inconsistencies and incomplete supplier reports further complicate the evaluation. To address these issues, fuzzy set theory [4] provides a valuable tool for modeling uncertainty and imprecise information. A recent extension of this theory, known as p,q-QOFSs [5], offers an advanced framework for capturing hesitation and imprecision in expert opinions. In this approach, the membership degree (MD) and non-membership degree (NMD) are raised to the powers of p and q, respectively, ensuring that their sum does not exceed one. The inclusion of the parameter p enhances the model's ability to represent incomplete information with greater flexibility and depth. Due to these advantages, p,q-QOFSs have been widely applied to various MCDM problems, demonstrating their effectiveness in handling complex and uncertain decision environments. Rahim *et al.* [6] developed cosine similarity and Euclidean distance measures for p,q-QOFSs and extended the TOPSIS model to address a company investment problem. Rahim *et al.* [7] developed an integrated decision-making model by combining COPRAS and CRITIC methods under p,q-quasirung orthopar fuzzy environment, then apply this model to green supplier selection. Alballa *et al.* [8] enhanced the CODAS approach to address multi-criteria group decision-making (MCGDM) challenges and utilized it to determine the optimal site for a retail store. Rahim *et al.* [9] proposed Dombi aggregation operators (AOs) within the p,q-ROF environment and introduced an MCGDM model utilizing these operators. Rahim *et al.* [10] proposed Bonferroni mean AOs within the p,q-ROF environment and developed an MCDM model incorporating these operators, which was then applied to stock selection for investment. Ali and Naeem [11] developed p,q-QOF Aczel–Alsina AOs for MCDM and applied them to evaluate the intensity of corruption. Arya and Pal [12] proposed an MCDM model using Jaccard and cosine similarity-driven AOs in an p,q-QOF environment, demonstrating its application in assessing government medical facilities across different Indian states.

In the context of MCGDM, the process of aggregating information is essential, as it involves combining the evaluations of several decision-experts (DEs) regarding alternatives under multiple criteria. The effectiveness of decision-making largely depends on the method used to aggregate individual opinions, preferences, or evaluations into a collective outcome. In this context, AOs serve as powerful mathematical tools designed to combine uncertain, imprecise, or fuzzy information in a systematic and meaningful manner. Among the various approaches to developing AOs, t-norms (TN) and t-conorms (TCN) are widely employed due to their ability to model conjunction and disjunction operations, respectively. A particularly flexible and effective class of TNs and TCNs is the Dombi TN and TCN [13], which are parameter-dependent functions. Their flexibility allows DEs to adjust the parameter values to control the strictness or permissiveness of the aggregation process, making them highly adaptable to different decision-making scenarios. By varying the parameter, the Dombi TN can mimic traditional TNs such as the minimum operator, product TN, or even more generalized forms, thus enhancing the versatility of the aggregation framework. Given their adaptability, Dombi-based AOs have been widely utilized in MCGDM models to enhance the precision and dependability of decision outcomes in various fuzzy environments. Some notable applications include Dombi AOs in intuitionistic fuzzy settings [14], linguistic Pythagorean fuzzy environments [15], Fermatean fuzzy contexts [16], interval-valued Fermatean fuzzy frameworks [17], q-rung orthopair fuzzy (q-ROF) environments [18], interval-valued spherical fuzzy environments [19], and hesitant fuzzy set environments [20], etc.

Again, existing Dombi AOs in different fuzzy environments, though effective, overlook variations in DEs' confidence levels (CLs), potentially leading to biased results. Their fixed mathematical formulations limit adaptability in uncertain decision settings. CL-based AOs address this by assigning greater weight to reliable information, enhancing accuracy and robustness in MCGDM. By incorporating CLs, decisions better reflect the credibility of expert opinions. Recognizing these advantages, researchers have introduced CL-based AOs in various fuzzy environments to enhance decision-making reliability and effectiveness. Seikh and Chatterjee [21] developed CL-based AOs under an IVSF environment and applied the MEREC-VIKOR approach for sustainable electric vehicle adoption. They also proposed CL-based Dombi AOs under the intuitionistic fuzzy framework and utilized them for the evaluation and selection of e-learning websites [22]. Chatterjee and Seikh [23] applied a CL-based decision-making framework in a q-rung orthopair picture fuzzy environment for municipal solid waste management. Joshi and Gegov [24] introduced CL based AOs under the q-rung orthopair fuzzy environment. Mahmood *et al.* [25] introduced CL based AOs within intuitionistic fuzzy rough sets for medical diagnosis. Lin *et al.* [26] investigated cost and profit evaluation by applying AOs on spherical fuzzy sets, incorporating CLs. Punetha and Komal [27] developed confidence picture fuzzy hybrid AOs for MCGDM. Khan *et al.* [28] introduced bipolar complex fuzzy AOs for decision-making.

The above analysis highlights the following gaps in the existing research:

- Existing fuzzy set-based approaches face challenges in effectively capturing hesitation and imprecise expert opinions in sustainable supplier selection. While the recently developed p,q-QROFSs offer greater flexibility in handling uncertainty, their application in this domain remains largely unexplored.
- In MCGDM, most AOs treat all expert opinions equally, failing to account for variations in reliability. This uniform treatment can introduce biases in decision-making, emphasizing the need for CL-based aggregation methods within the p,q-QOF framework.
- Although Dombi AOs offer flexible information fusion capabilities, their integration with p,q-QROFSs has not been thoroughly investigated. Exploring these operators can enhance the accuracy and robustness of decision-making processes in uncertain environments.

The motivation of the study is given as follows:

- The automotive industry is increasingly prioritizing sustainable procurement strategies that align with environmental, social, and economic goals. Selecting a sustainable supplier requires a decision-making framework that integrates multiple criteria while addressing uncertainty in expert evaluations.
- Conventional decision-making methods frequently face challenges in managing uncertain and imprecise information, which can result in inconsistencies during supplier assessment. A robust model incorporating advanced fuzzy logic and aggregation techniques is essential for ensuring more reliable and well-informed decisions.
- The p,q-QOFS offers a highly adaptable framework for handling uncertain and imprecise information, making it well-suited for addressing complex decision-making problems.
- Existing Dombi AOs do not consider the CLs of DEs, which are crucial for refining decision accuracy. This research introduces CL-based Dombi AOs under p,q-QOFS, improving the reliability of information fusion in MCGDM.

The important contributions to this study are listed below.

- This study introduces Dombi weighted averaging and geometric AOs under p,q-QOFSs. These operators incorporate CLs, ensuring expert reliability influences decision-making outcomes.
- An improved MCGDM model using CL-based Dombi AOs under p,q-QOFSs is developed. It enhances decision accuracy by addressing uncertainty and varying CLs in expert evaluations.
- The proposed model is applied to evaluating and selecting sustainable suppliers in the automotive industry, ensuring more reliable and data-driven procurement decisions.
- A comparative study with existing methods highlights the superiority of the proposed approach. Sensitivity analysis confirms its robustness by assessing stability under varying parameter values.

The structure of this study is outlined as follows: Section 2 outlines the essential preliminaries. Section 3 describes the development of CL-based Dombi AOs for p,q-QOFSs and discusses their fundamental properties. Section 4 explains the MCGDM framework that incorporates the proposed Dombi AOs within the context of p,q-QOFSs. In Section 5, the effectiveness of the proposed approach is illustrated through a real-world example focusing on sustainable supplier selection in the automotive industry. Lastly, Section 6 summarizes the findings and highlights the study's limitations along with suggestions for future research.

2. Preliminaries

This section presents essential definitions and preliminary concepts. In the entire paper, the universal set is denoted by X.

Definition 2.1 *[5] For two positive integers p and q, a p,q-QOFS β defined on X can be expressed as*

$$\beta = \{\langle c, M_\beta(c), N_\beta(c)\rangle | c \in X\}$$

where $M_\beta : X \longrightarrow [0,1]$ and $N_\beta : X \longrightarrow [0,1]$ represents the MD and NMD of $c \in X$. These functions satisfy the condition $(M_\beta(c))^p + (N_\beta(c))^q \leq 1$. For convenience, the pair $\beta = (M_\beta, N_\beta)$ is referred to as a p,q-QOF number (p,q-QOFN).

Definition 2.2 *[5] Let $\beta = (M_\beta, N_\beta)$, $\beta_1 = (M_{\beta_1}, N_{\beta_1})$ and $\beta_2 = (M_{\beta_2}, N_{\beta_2})$ be the p,q-QOFNs and $\xi > 0$. Then*

1. $\beta_1 \bigoplus \beta_2 = \left(\sqrt[p]{(M_{\beta_1})^p + (M_{\beta_2})^p - (M_{\beta_1})^p (M_{\beta_2})^p}, N_{\beta_1} N_{\beta_2}\right);$
2. $\beta_1 \bigotimes \beta_2 = \left(M_{\beta_1} M_{\beta_2}, \sqrt[q]{(N_{\beta_1})^q + (N_{\beta_2})^q - (N_{\beta_1})^q (N_{\beta_2})^q}\right);$
3. $\beta^\xi = \left((M_\beta)^\xi, \sqrt[q]{1 - (1 - (N_\beta)^q)^\xi}\right);$
4. $\xi\beta = \left(\sqrt[p]{1 - (1 - (M_\beta)^p)^\xi}, (N_\beta)^\xi\right).$

Definition 2.3 *[5] The score function Φ and the accuracy function Ψ for the p,q-QOFN $\beta = (M_\beta, N_\beta)$ are defined as*

$$\Phi(\beta) = \frac{1 + (M_\beta)^p - (N_\beta)^q}{2} \tag{1}$$

$$\Psi(\beta) = (M_\beta)^p + (N_\beta)^q \tag{2}$$

where $0 \leq \Phi(\beta) \leq 1$ and $0 \leq \Psi(\beta) \leq 1$.
Let $\beta_1 = (M_{\beta_1}, N_{\beta_1})$ and $\beta_2 = (M_{\beta_2}, N_{\beta_2})$ be two p,q-QOFNs, then

- $\beta_1 \succ \beta_2$ *if* $\Phi(\beta_1) > \Phi(\beta_2)$;
- $\beta_2 \succ \beta_1$ *if* $\Phi(\beta_1) < \Phi(\beta_2)$;
- *If* $\Phi(\beta_1) = \Phi(\beta_2)$, *then*
  - $\beta_1 \succ \beta_2$ *if* $\Psi(\beta_1) > \Psi(\beta_2)$;
  - $\beta_2 \succ \beta_1$ *if* $\Psi(\beta_1) < \Psi(\beta_2)$;

- $\beta_1 \sim \beta_2$ if $\Psi(\beta_1) = \Psi(\beta_2)$.

Definition 2.4 *[13] For any two real numbers r and s, DTN (T_D) and DTCN (S_D) are defined as*

$$T_D(r,s) = \frac{1}{1+[(\frac{1-r}{r})^\eta + (\frac{1-s}{s})^\eta]^{\frac{1}{\eta}}} \text{ and } S_D(r,s) = 1 - \frac{1}{1+[(\frac{r}{1-r})^\eta + (\frac{s}{1-s})^\eta]^{\frac{1}{\eta}}}$$

where $\eta > 0$ is the adjustable operational parameter of the Dombi functions, and both r and s are elements of the interval $[0,1]$.

Definition 2.5 *[9] Let $\beta = (M_\beta, N_\beta)$, $\beta_1 = (M_{\beta_1}, N_{\beta_1})$ and $\beta_2 = (M_{\beta_2}, N_{\beta_2})$ be three p,q-QOFNs and $\xi > 0$. Then Dombi operation within p,q-QOFNs are defined as follows.*

1. $\beta_1 \bigoplus \beta_2 = \left(\sqrt[p]{1 - \frac{1}{1+\{\left(\frac{(M_{\beta_1})^p}{1-(M_{\beta_1})^p}\right)^\eta + \left(\frac{(M_{\beta_2})^p}{1-(M_{\beta_2})^p}\right)^\eta\}^{\frac{1}{\eta}}}}, \sqrt[q]{\frac{1}{1+\{\left(\frac{1-(N_{\beta_1})^q}{(N_{\beta_1})^q}\right)^\eta + \left(\frac{1-(N_{\beta_2})^q}{(N_{\beta_2})^q}\right)^\eta\}^{\frac{1}{\eta}}}}\right)$.
2. $\beta_1 \bigotimes \beta_2 = \left(\sqrt[p]{\frac{1}{1+\{\left(\frac{1-(M_{\beta_1})^p}{(M_{\beta_1})^p}\right)^\eta + \left(\frac{1-(M_{\beta_2})^p}{(M_{\beta_2})^p}\right)^\eta\}^{\frac{1}{\eta}}}}, \sqrt[q]{1 - \frac{1}{1+\{\left(\frac{(N_{\beta_1})^q}{1-(N_{\beta_1})^q}\right)^\eta + \left(\frac{(N_{\beta_2})^q}{1-(N_{\beta_2})^q}\right)^\eta\}^{\frac{1}{\eta}}}}\right)$.
3. $\xi\beta = \left(\sqrt[p]{1 - \frac{1}{1+\{\xi\left(\frac{(M_\beta)^p}{1-(M_\beta)^p}\right)^\eta\}^{\frac{1}{\eta}}}}, \sqrt[q]{\frac{1}{1+\{\xi\left(\frac{1-(N_\beta)^q}{(N_\beta)^q}\right)^\eta\}^{\frac{1}{\eta}}}}\right)$.
4. $\beta^\xi = \left(\sqrt[p]{\frac{1}{1+\{\xi\left(\frac{1-(M_\beta)^p}{(M_\beta)^p}\right)^\eta\}^{\frac{1}{\eta}}}}, \sqrt[q]{1 - \frac{1}{1+\{\xi\left(\frac{(N_\beta)^q}{1-(N_\beta)^q}\right)^\eta\}^{\frac{1}{\eta}}}}\right)$.

Definition 2.6 *[9] The aggregated result of the p,q-QOFNs $\beta_k = (M_{\beta_k}, N_{\beta_k})(k = 1, 2, \ldots, n)$ using the p,q-QOF Dombi weighted averaging (p,q-QOFDWA) and p,q-QOF Dombi weighted geometric (p,q-QOFDWG) operators are given as follws:*

$$p,q - QOFDWA(\beta_1, \beta_2, \ldots, \beta_n) = \left(\sqrt[p]{1 - \frac{1}{1+\sum_{k=1}^n\{\xi_k(\frac{(M_{\beta_k})^p}{1-(M_{\beta_k})^p})^\eta\}^{\frac{1}{\eta}}}}, \sqrt[q]{\frac{1}{1+\sum_{k=1}^n\{\xi_k(\frac{1-(N_{\beta_k})^q}{(N_{\beta_k})^q})^\eta\}^{\frac{1}{\eta}}}}\right)$$

$$p,q - QOFDWG(\beta_1, \beta_2, \ldots, \beta_n) = \left(\sqrt[p]{\frac{1}{1+\sum_{k=1}^n\{\xi_k(\frac{1-(M_{\beta_k})^p}{(M_{\beta_k})^p})^\eta\}^{\frac{1}{\eta}}}}, \sqrt[q]{1 - \frac{1}{1+\sum_{k=1}^n\{\xi_k(\frac{(N_{\beta_k})^q}{1-(N_{\beta_k})^q})^\eta\}^{\frac{1}{\eta}}}}\right).$$

Here, ξ_k be the weight of β_k satisfying the conditions $\xi_k>0$ and $\sum_{k=1}^n \xi_k = 1$.

Definition 2.7 *[29] The aggregated result of the p,q-QOFNs $\beta_k = (M_{\beta_k}, N_{\beta_k})(k = 1, 2, \ldots, n)$ using the p,q-QOF weighted averaging (p,q-QOFWA) operator is given as follws:*

$$p,q - QOFWA(\beta_1, \beta_2, \ldots, \beta_n) = \left(\sqrt[p]{1 - \prod_{k=1}^n (1 - M_{\beta_k}^p)^{\xi_k}}, \prod_{k=1}^n (N_{\beta_k})^{\xi_k}\right). \tag{3}$$

Here, ξ_k be the weight of β_k satisfying the conditions $\xi_k>0$ and $\sum_{k=1}^n \xi_k = 1$.

3. CL-based Dombi AOs for p,q-QOFSs

This section introduces the formulation of Dombi AOs in the p,q-QOF framework, integrating the concept of CLs.

Definition 3.1 *The aggregated value of the p,q-QOFNs $\beta_k = (M_{\beta_k}, N_{\beta_k})(k = 1, 2 \ldots, n)$ using the CL based p,q-QOF Dombi weighted averaging/geometric (CLp,q-QOFDWA/CLp,q-QOFDWG) AOs are given by*

$$CLp,q-QOFDWA(\beta_1, \beta_2, \ldots, \beta_n) = \bigoplus_{k=1}^{n} (\xi_k l_k)\beta_k.$$

$$CLp,q-QOFDWG(\beta_1, \beta_2, \ldots, \beta_n) = \bigotimes_{k=1}^{n} (\beta_k)^{\xi_k l_k}.$$

where $l_k (0 \le l_k \le 1)$ be the confidence level β_i, and ξ_i's are given in Definition 2.6.

Theorem **3.1** *If $\beta_k = (M_{\beta_k}, N_{\beta_k})(k = 1, 2 \ldots, n)$ be a collection of p,q-QOFNs and l_k be the CL of β_k then the aggregated value using CLp,q-QOFDWA (CLp,q-QOFDWG) operators are given by*

$$CLp,q-QOFDWA(\beta_1, \beta_2, \ldots, \beta_n) = \left(\sqrt[p]{1 - \frac{1}{1 + [\sum_{k=1}^{n} \{l_k \xi_k (\frac{(M_{\beta_k})^p}{1-(M_{\beta_k})^p})^\eta\}]^{\frac{1}{\eta}}}}, \sqrt[q]{\frac{1}{1 + [\sum_{k=1}^{n} \{l_k \xi_k (\frac{1-(N_{\beta_k})^q}{(N_{\beta_k})^q})^\eta\}]^{\frac{1}{\eta}}}} \right)$$

$$CLp,q-QOFDWG(\beta_1, \beta_2, \ldots, \beta_n) = \left(\sqrt[p]{\frac{1}{1 + [\sum_{k=1}^{n} \{l_k \xi_k (\frac{1-(M_{\beta_k})^p}{(M_{\beta_k})^p})^\eta\}]^{\frac{1}{\eta}}}}, \sqrt[q]{1 - \frac{1}{1 + [\sum_{k=1}^{n} \{l_k \xi_k (\frac{(N_{\beta_k})^q}{1-(N_{\beta_k})^q})^\eta\}]^{\frac{1}{\eta}}}} \right).$$

Proof: For two p,q-QOFNs β_1 and β_2, we have

$$CLp,q-QOFDWA(\beta_1, \beta_2) = \bigoplus_{k=1}^{2} (\xi_k l_k)\beta_k =$$

$$\left(\sqrt[p]{1 - \frac{1}{1+\{\xi_1 l_1 \left(\frac{(M_{\beta_1})^p}{1-(M_{\beta_1})^p}\right)^\eta\}^{\frac{1}{\eta}}}}, \sqrt[q]{\frac{1}{1+\{\xi_1 l_1 \left(\frac{1-(N_{\beta_1})^q}{(N_{\beta_1})^q}\right)^\eta\}^{\frac{1}{\eta}}}} \right) \oplus$$

$$\left(\sqrt[p]{1 - \frac{1}{1+\{\xi_2 l_2 \left(\frac{(M_{\beta_2})^p}{1-(M_{\beta_2})^p}\right)^\eta\}^{\frac{1}{\eta}}}}, \sqrt[q]{\frac{1}{1+\{\xi_2 l_2 \left(\frac{1-(N_{\beta_2})^q}{(N_{\beta_2})^q}\right)^\eta\}^{\frac{1}{\eta}}}} \right)$$

$$= \left(\sqrt[p]{1 - \frac{1}{1+\left(\xi_1 l_1 \left(\frac{(M_{\beta_1})^p}{1-(M_{\beta_1})^p}\right)^\eta + \xi_2 l_2 \left(\frac{(M_{\beta_2})^p}{1-(M_{\beta_2})^p}\right)^\eta\right)^{\frac{1}{\eta}}}}, \sqrt[q]{\frac{1}{1+\left(\xi_1 l_1 \left(\frac{1-(N_{\beta_1})^q}{(N_{\beta_1})^q}\right)^\eta + \xi_2 l_2 \left(\frac{1-(N_{\beta_2})^q}{(N_{\beta_2})^q}\right)^\eta\right)^{\frac{1}{\eta}}}} \right)$$

$$= \left(\sqrt[p]{1 - \frac{1}{1+\sum_{k=1}^{2} \{l_k \xi_k (\frac{(M_{\beta_k})^p}{1-(M_{\beta_k})^p})^\eta\}^{\frac{1}{\eta}}}}, \sqrt[q]{\frac{1}{1+\sum_{k=1}^{2} \{l_k \xi_k (\frac{1-(N_{\beta_k})^q}{(N_{\beta_k})^q})^\eta\}^{\frac{1}{\eta}}}} \right).$$

Hence, the theorem holds true for the case when $k = 2$.

Now, suppose that the statement is true for $k = m$, where m is an arbitrary natural number.

Thus, $CLp,q-QOFDWA(\beta_1, \beta_2, \ldots, \beta_m) = \left(\sqrt[p]{1 - \frac{1}{1+[\sum_{k=1}^{m} \{l_k \xi_k (\frac{(M_{\beta_k})^p}{1-(M_{\beta_k})^p})^\eta\}]^{\frac{1}{\eta}}}}, \sqrt[q]{\frac{1}{1+[\sum_{k=1}^{m} \{l_k \xi_k (\frac{1-(N_{\beta_k})^q}{(N_{\beta_k})^q})^\eta\}]^{\frac{1}{\eta}}}} \right).$

Now,

$$CLp,q-QOFDWA(\beta_1, \beta_2, \ldots, \beta_m, \beta_{m+1}) = \bigoplus_{k=1}^{m+1} (\xi_k l_k)\beta_k =$$

$$\left(\sqrt[p]{1-\frac{1}{1+[\sum\limits_{k=1}^{m}\{l_k\xi_k(\frac{(M_{\beta_k})^p}{1-(M_{\beta_k})^p})^\eta\}]^{\frac{1}{\eta}}}}, \sqrt[q]{\frac{1}{1+[\sum\limits_{k=1}^{m}\{l_k\xi_k(\frac{1-(N_{\beta_k})^q}{(N_{\beta_k})^q})^\eta\}]^{\frac{1}{\eta}}}}\right)\oplus$$
$$\left(\sqrt[p]{1-\frac{1}{1+\{\xi_{m+1}l_{m+1}\left(\frac{(M_{\beta_{m+1}})^p}{1-(M_{\beta_{m+1}})^p}\right)^\eta\}^{\frac{1}{\eta}}}}, \sqrt[q]{\frac{1}{1+\{\xi_{m+1}l_{m+1}\left(\frac{1-(N_{\beta_{m+1}})^q}{(N_{\beta_{m+1}})^q}\right)^\eta\}^{\frac{1}{\eta}}}}\right)$$
$$=\left(\sqrt[p]{1-\frac{1}{1+\left(\sum\limits_{k=1}^{m}l_k\xi_k(\frac{(M_{\beta_k})^p}{1-(M_{\beta_k})^p})^\eta+l_{m+1}\xi_{m+1}(\frac{(M_{\beta_{m+1}})^p}{1-(M_{\beta_{m+1}})^p})^\eta\right)^{\frac{1}{\eta}}}}, \sqrt[q]{\frac{1}{1+\left(\sum\limits_{k=1}^{m}l_k\xi_k(\frac{1-(N_{\beta_k})^q}{(N_{\beta_k})^q})^\eta+l_{m+1}\xi_{m+1}(\frac{1-(N_{\beta_{m+1}})^q}{(N_{\beta_{m+1}})^q})^\eta\right)^{\frac{1}{\eta}}}}\right)$$
$$=\left(\sqrt[p]{1-\frac{1}{1+\sum\limits_{k=1}^{m+1}\{l_k\xi_k(\frac{(M_{\beta_k})^p}{1-(M_{\beta_k})^p})^\eta\}^{\frac{1}{\eta}}}}, \sqrt[q]{\frac{1}{1+\sum\limits_{k=1}^{m+1}\{l_k\xi_k(\frac{1-(N_{\beta_k})^q}{(N_{\beta_k})^q})^\eta\}^{\frac{1}{\eta}}}}\right).$$

Thus, if the theorem holds for $k=m$, it must also hold for $k=m+1$. Hence, by applying the principle of mathematical induction, the theorem is proven to hold true for all k.

Similarly, the aggregated result obtained through the CLp,q-QOFDWG operator can be derived following the same approach.

Subsequently, we present several desirable properties of the proposed CLp,q-QOFDWA and CLp,q-QOFDWG operators.

Property 3.1 *If all the p,q-QOFNs $\beta_i=(M_{\beta_i},N_{\beta_i}),(i=1,2,\dots,n)$ are equal to $\beta=(M_\beta,N_\beta)$, then $CLp,q-QOFDWA(\beta_1,\beta_2,\dots,\beta_n)=I$ and $CLp,q-QOFDWG(\beta_1,\beta_2,\dots,\beta_n))=I$.*

Property 3.2 *Consider a set of p,q-QOFNs denoted by $\beta_i=(M_{\beta_i},N_{\beta_i})(i=1,2,\dots,n)$, each associated with a confidence level $l_i(0\le l_i\le 1)$. Let, $\beta^-=\min_i\{\beta_i\}$ and $\beta^+=\max_i\{\beta_i\}$. Then,*

$$\beta^-\le CLp,q-QOFDWA(\beta_1,\beta_2,\dots,\beta_n)\le\beta^+$$

$$\beta^-\le CLp,q-QOFDWG(\beta_1,\beta_2,\dots,\beta_n)\le\beta^+$$

where $min\{\beta_i\}=(min(M_{\beta_i}),max(N_{\beta_i})),max\{\beta_i\}=(max(M_{\beta_i}),min(N_{\beta_i})),\forall i.$

Property 3.3 *Let $\{\beta_i|i=1(1)n\}$ and $\{\beta_i^*|i=1(1)n\}$ be two families of p,q-QOFNs such that $\beta_i\preceq{\beta_i}^*$ then $CLp,q-QOFDWA(\beta_1,\beta_2,\dots,\beta_n)\le CLp,q-QOFDWA(\beta_1^*,\beta_2^*,\dots,\beta_n^*)$ and $CLp,q-QOFDWG(\beta_1,\beta_2,\dots,\beta_n)\le CLp,q-QOFDWG(\beta_1^*,\beta_2^*,\dots,\beta_n^*)$.*

4. MCGDM Model using the Proposed Operators

In this section, we develop a MCGDM model using the proposed CLp,q-QOFDWA and CLp,q-QOFDWG operators.

This approach involves three key stages. The first step consists in collecting data from multiple experts for each available alternative related to the attributes, which is organized into matrices. The next phase involves combining the decision matrices gathered from various experts. Finally, alternatives are evaluated and prioritized according to the score values derived from the aggregated data.

Consider the set $\{K_1,K_2,\dots,K_m\}$ representing m alternatives and the set $\{R_1,R_2,\dots,R_n\}$ representing n criteria. The weights corresponding to these criteria are denoted as $\xi_1,\xi_2,\dots,\xi_n$, which fulfill the conditions $\sum\limits_{k=1}^{n}\xi_k=1$ and $0\le\xi_k\le 1$ for each n. Let us consider a group of i experts, $D_1,D_2,\dots,D_i$, who evaluate a set of alternatives based on multiple criteria. Using the p,q-QOFN, each expert assesses each alternative and provides their evaluations in the form of p,q-QOFNs as, $\gamma_{cd}^r=(M_{\beta_{cd}}^r,N_{\beta_{cd}}^r)$, where $c=1,2,\dots,m;d=1,2,\dots,n$ and $r=1,2,\dots,k$, and

$(M^r_{\beta_{cd}})^p + (N^r_{\beta_{cd}})^q \leq 1$. The experts also specify the extent of their familiarity with the given alternatives by assigning confidence levels l^k_{cd}, where $0 \leq l^k_{cd} \leq 1$, thereby incorporating the notion of confidence into the evaluation process. Also assume that experts weights are $\alpha_1, \alpha_2, \ldots, \alpha_i$, where $0 \leq \alpha_r \leq 1$ and $\sum_{r=1}^{i} \alpha_r = 1$.

The proposed MCGDM framework, which uses CLp,q-QOFDWA and CLp,q-QOFDWG AOs, follows these steps.

Step 1. Identify the problem, select the experts along with their confidence levels, and define the alternatives and criteria associated with the given problem.

Step 2. The weight of each decision expert is computed using Eq. (4). Let α_r denote the weight of the r^{th} expert, then

$$\alpha_r = \frac{y_r}{\sum_{r=1}^{i} y_r}, \tag{4}$$

where y_r represents the number of years of experience of the r^{th} expert.

Step 3. Computation of criteria weights using the AHP method [30]. The computaion procedure comprised of the following steps:

Step 3.1. Construct the pairwise comparison matrix by comparing each criteria with the others using the scale of importance presented in Table 1. Let $R = (r_{ab})_{n\times n}$ be the pairwise comparison matrix.

Table 1
AHP relative importance scale

Scale Value	Explanation
1	Criteria are equally important
3	One criterion is moderately more important than the other
5	One criterion is strongly more important
7	One criterion is very strongly more important
9	One criterion is extremely more important
2,4,6,8	Intermediate values

Step 3.2. Normalize the pairwise comparison matrix using Eq. (5). Let $R' = (r'_{ab})_{n\times n}$ be the normalized pairwise comparison matrix, where

$$r'_{ab} = \frac{r_{ab}}{\sum_{a=1}^{n} r_{ab}}. \tag{5}$$

Step 3.3. Compute the final weights of the criteria using Eq. (6). Let ξ_a be the weight of the k^{th} criteria. Then

$$\xi_a = \frac{1}{n}\sum_{b=1}^{n} r'_{ab}. \tag{6}$$

Step 4. Gather the decision matrices expressed in linguistic variables (LVs) together with their level of confidence. Then, Transform the LVs assigned by the decision experts (DEs) into p,q-QOFNs using the predefined linguistic scale presented in Table 2. This scale is constructed based on the semantics and relative strength of each linguistic term, where each LV is mapped to a p,q-QOFN of the form (μ, ν), where μ and ν denote the membership and non-membership degrees respectively, satisfying the condition $\mu^p + \nu^q \leq 1$ for the chosen values of p and q. This results in the p,q-QOF decision matrix with their level of confidence $M_r = \langle \gamma^r_{cd}, l^r_{cd} \rangle_{m \times n}$ for $r = 1, 2, \ldots, i$, where $c = 1, 2, \ldots, m$ and $d = 1, 2, \ldots, n$.

Table 2
DEs' confidence level for each attribute

LVs	p,q-QOFNs
Very Bad (VB)	(0.10,0.98)
Bad (B)	(0.25,0.75)
Medium (M)	(0.45,0.55)
Good(G)	(0.75,0.25)
Very Good (VB)	(0.98,0.10)

In Table 2, a VB evaluation implies high uncertainty and very low satisfaction, which is reflected by assigning a low membership degree and a high non-membership degree, such as (0.20, 0.98). Similarly, 'B' indicates dissatisfaction with slightly more positivity than VB, represented by (0.50, 0.96), 'M' shows neutrality or hesitation, 'G' implies a high level of agreement, and 'VG' represents strong satisfaction.

Step 5. Determine the aggregated decision matrix by applying the CLs and weights of the decision experts, utilizing the CLp,q-QOFDWA (or CLp,q-QOFDWG) AO given in Definition 3.1. Consider the aggregated decision matrix $\overline{M} = (\overline{\gamma}_{cd})_{m \times n}$, where each element is given by $\overline{\gamma}_{cd} = (M_{\beta_{cd}}, N_{\beta_{cd}})$ and calculated using Eq. (7).

$$\overline{\gamma}_{cd} = (M_{\beta_{cd}}, N_{\beta_{cd}}) = \left(\sqrt[p]{1 - \frac{1}{1 + [\sum_{r=1}^{i} \{ l^r_{cd} \alpha_r (\frac{(M^r_{\beta_{cd}})^p}{1-(M^r_{\beta_{cd}})^p})^\eta \}]^{\frac{1}{\eta}}}}, \sqrt[q]{\frac{1}{1 + [\sum_{r=1}^{i} \{ l^r_{cd} \alpha_r (\frac{1-(N^r_{\beta_{cd}})^q}{(N^r_{\beta_{cd}})^q})^\eta \}]^{\frac{1}{\eta}}}} \right). \tag{7}$$

Step 6. Compute the combined value for each alternative using the p,q-QOFWA operator as defined in Definition 2.7. Then, aggregate the overall value of each alternative, denoted as ρ_c, using Eq. (8).

$$\rho_c = \left(\sqrt[p]{1 - \prod_{d=1}^{n} (1 - M^p_{\beta_{cd}})^{\xi_d}}, \prod_{d=1}^{n} (N_{\beta_{cd}})^{\xi_d} \right). \tag{8}$$

Step 7. Determine the score value $\Phi(\rho_c)(c = 1(1)m)$ utilizing Eq. (1) for each combined value ρ_c.

Step 8. If a particular alternative achieves the highest score among all options, it is considered the most preferred choice. Thus, alternative K_c deemed the best if $\Phi(\alpha_c) = \max_{1 \leq s \leq m} \{\Phi(\alpha_s)\}$. However, if no unique alternative holds the highest score, proceed to the next step.

Step 9. In cases where multiple alternatives attain the same score, Eq. (2) should be used to calculate the accuracy function Ψ for these alternatives. Then:

- If several alternatives share the highest score, the final selection is based on the one with the maximum $\Psi-$functional value.
- If multiple alternatives also have identical functional values, any of them can be regarded as an acceptable choice.

Figure 1 illustrates the step of the proposed MCGDM approach.

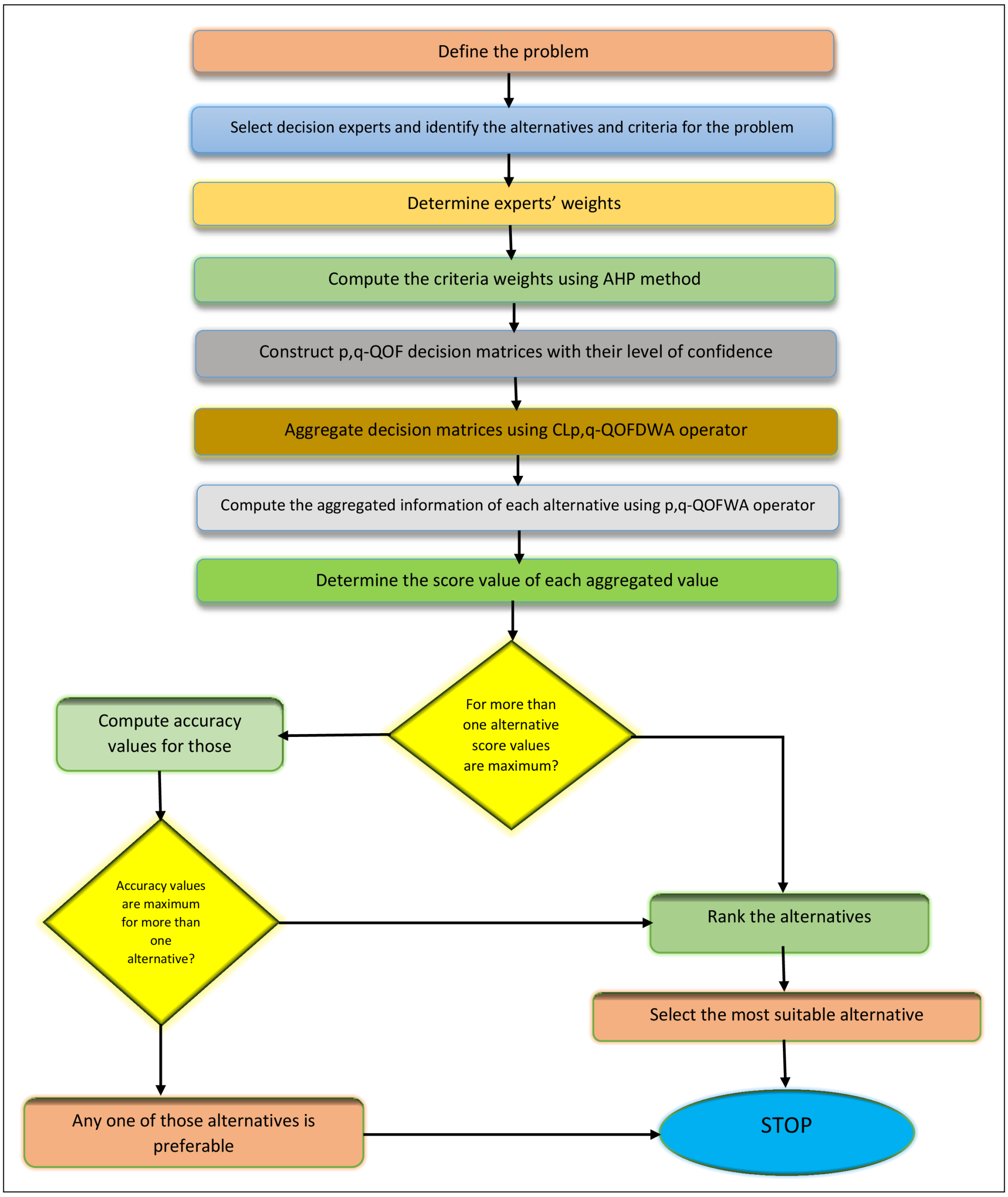

Fig. 1. Framework of the proposed model

5. Sustainable Supplier Selection in the Automotive Industry

Sustainable supplier selection is a crucial process in the automotive industry, as it directly impacts environmental responsibility, economic efficiency, and social welfare. Automotive companies are increasingly expected to go beyond traditional cost and quality considerations by incorporating sustainability into their supplier evaluation and selection processes. This ensures long-term value creation for businesses, enhances brand reputation, and contributes to global sustainability goals.

To make informed decisions, automotive companies need to evaluate suppliers based on multiple sustainability criteria. These criteria generally span environmental, economic, and social dimensions, each playing a significant role in responsible supply chain management. However, selecting the most appropriate supplier is a complex task due to conflicting objectives, uncertainty in data, and subjective expert judgments. To overcome these challenges, researchers and practitioners have developed a variety of advanced decision-making models.

5.1 Review of Decision-Making Models for Supplier Selection in the Automotive Industry

In response to the growing complexity of supplier selection decisions, the literature presents severalMCDM models that integrate fuzzy logic, and hybrid approaches. These models aim to handle imprecise information and balance multiple sustainability dimensions effectively.

For instance, Tronnebati *et al.* [31] developed a comprehensive decision-making framework that integrates AHP, TOPSIS, and WASPAS techniques within a fuzzy environment. They effectively utilized this approach to address the problem of selecting green suppliers in Morocco's automotive sector. Kara *et al.* [32] developed a MCDM model based on Dempster-Shafer theory to select green supplier in the automotive industry. Bas [33] proposed a combined decision-making framework that merges the AHP and TOPSIS techniques under an interval type-2 fuzzy environment. This integrated approach was effectively utilized to address green supplier selection in the automotive sector. Gergin *et al.* [34] proposed a combined decision-making approach integrating DEMATEL and TOPSIS techniques under an intuitionistic fuzzy setting to facilitate supplier selection in the automotive sector. Bah and Tulkinov [35] integrated the ordinal priority approach with the TOPSIS method to develop a decision-making model, which they applied for the evaluation of automotive parts suppliers. Dweiri *et al.* [36] demonstrated the application of AHP-based MCDM frameworks through a case study in the Pakistani automotive sector. Dang *et al.* [37] developed a two-stage MCDM framework that combines AHP and COPRAS techniques within a spherical fuzzy environment to effectively address supplier selection in the Vietnamese automobile industry.

Although these models have significantly advanced supplier evaluation methods, most existing approaches do not incorporate the CL of DEs, which can play a vital role in reflecting the reliability of expert judgments. Practical decision-making scenarios require not only sophisticated mathematical models but also clear problem formulation and expert-driven analysis that account for real-world constraints, varying expertise, and the certainty associated with expert evaluations. Therefore, this study builds upon previous models by integrating CLs into the decision-making process, involving expert judgment along with realistic supplier profiles. This integration demonstrates a more structured and reliable approach for sustainable supplier selection.

5.2 Problem Statement

In this study, three DEs $D1, D2$ and $D3$ are engaged to evaluate five potential suppliers based on key sustainability criteria. Each expert brings a distinct background and area of expertise to ensure a balanced and informed evaluation. Expert $D1$ has 15 years of experience in supply chain management, specializing in sustainable procurement. Expert $D2$ has 12 years of experience in environmental compliance, focusing on carbon footprint reduction. Expert $D3$ has 10 years of experience in corporate social responsibility, with an emphasis on labor rights and ethical procurement.

The five suppliers considered in this evaluation are Supplier $K1,$ a well-established supplier with strong environmental policies but higher costs; Supplier $K2,$ a cost-effective supplier with moderate sustainability practices; Supplier $K3,$ a new supplier with innovative green technologies but limited operational history; Supplier $K4,$ a regional supplier with reliable delivery performance and average environmental compliance; and Supplier $K5,$ a global supplier offering a wide product range with variable sustainability records across locations. The DEs will evaluate these suppliers based on three sustainability criteria: environmental, economic, and social.

The environmental criteria include:

- Carbon emissions (tons per year) $(R1)$ [38]: This criterion measures the total greenhouse gas emissions generated by the supplier's manufacturing and logistics processes. Lower emission levels indicate a more environmentally conscious supplier, reducing its contribution to global climate change. The automotive industry is known for significant CO_2 emissions, making this metric crucial for assessing environmental responsibility. Suppliers that adopt cleaner technologies, electric vehicles for transportation, and green energy sources such as solar or wind power are rated more favorably. Additionally, consistent monitoring and reporting of emissions show a proactive approach toward environmental regulations and sustainability goals.

- Waste management $(R2)$ [38, 39]: This criterion evaluates the percentage of waste materials that are recycled or disposed of through environmentally responsible methods. A higher recycling efficiency indicates stronger sustainability efforts by reducing landfill dependency and fostering a circular economy. Suppliers with robust waste management policies demonstrate a commitment to environmental stewardship and regulatory compliance. Moreover, effective waste utilization in production processes can lead to reduced material costs, improved operational efficiency, and a positive brand image in environmentally conscious markets.

- Energy consumption (kWh/unit produced) $(R3)$ [38]: This criterion measures the total energy consumption per unit of product manufactured. Lower energy consumption per unit indicates higher energy efficiency and contributes to a reduced carbon footprint. Suppliers that invest in renewable energy sources, such as solar or wind power, are evaluated more favorably. Additionally, continuous improvements in energy-efficient technologies can enhance competitiveness, reduce operational expenses, and align with global sustainability goals, making such suppliers more attractive for long-term partnerships.

The economic criteria include:

- Cost competitiveness $(R4)$ [38, 40]: This criterion evaluates the supplier's pricing structure in comparison to industry standards. A competitive cost, without compromising product quality, is essential for sustainable financial performance and overall business growth. Lower procurement costs enable companies to remain profitable while maintaining affordability for end customers. Additionally, suppliers who offer stable, transparent, and reasonable pricing structures contribute to financial predictability and effective budget planning. Establishing partnerships with such suppliers ensures a reliable, long-term business relationship and supports strategic decision-making for future investments.

- Financial stability $(R5)$ [38, 40]: A financially stable supplier ensures reliability and minimal risk of business failure or supply chain disruptions. Higher credit ratings indicate a stronger financial position and the ability to fulfill long-term contracts. Suppliers with better financial health are more likely to invest in sustainability initiatives. Evaluating financial reports and industry benchmarks helps in assessing their fiscal responsibility. Additionally, financial stability reflects the supplier's ability to adapt to market fluctuations and unforeseen challenges without compromising performance.

- Delivery performance (on-time delivery rate %) $(R6)$ [38, 40]: This criterion measures the supplier's ability to meet delivery deadlines consistently. A high on-time delivery rate ensures smooth production planning and avoids supply chain delays. Suppliers with robust logistics and inventory management systems score higher in this metric. Efficient delivery performance reduces inventory holding costs and production halts. Moreover, it reflects the supplier's reliability and commitment, which directly impacts the overall operational efficiency and customer satisfaction.

The social criteria include:

- Labor conditions (worker satisfaction index) $(R7)$ [38, 40]: This assesses worker satisfaction based on fair wages, workplace safety, and labor policies. Companies with better working conditions retain skilled employees and maintain higher productivity. Suppliers adhering to international labor laws and ethical employment practices rank higher. Satisfied workers contribute to a healthier work environment and lower turnover rates. Moreover, a positive labor environment enhances the company's reputation and fosters long-term business sustainability.

- Corporate social responsibility (CSR) activities $(R8)$ [38, 40]: This measures the supplier's investment in community development and social welfare initiatives. Companies actively engaging in CSR contribute to local economic development and environmental conservation. Higher CSR investment indicates a commitment to ethical business practices and corporate sustainability. Suppliers with social responsibility programs tend to have a positive brand reputation and stronger stakeholder relationships.

The hierarchy of the selection of sustainable suppliers is presented in Figure 2.

5.3 Solution Procedure

Step 1. In this decision-making problem, five alternatives ($K1, K2, \ldots, K5$) are evaluated based on eight criteria ($R1, R2, \ldots, R8$). The evaluation is carried out by three decision experts ($D1$, $D2$, $D3$), possessing 15, 12, and 10 years of experience, respectively.

Step 2. The weights of the decision experts based on their years of experience are calculated using Eq. (4) as follows: $\alpha_1 = \frac{15}{37} = 0.406, \alpha_2 = \frac{12}{37} = 0.324, \alpha_3 = \frac{10}{37} = 0.270.$

Step 3. Computation of experts weights using AHP method. Here we compute the experts weights using the following steps.

Step 3.1. Using the scale of importance provided in Table 1, the pairwise comparison matrix for the eight criteria $R_1, R_2, \ldots, R_8$ is presented in Table 3.

Step 3.2. The normalized pairwise comparison matrix, computed using Eq. (5), is presented in Table 4.

Step 3.3. Using Eq. (6), the normalized weights of the eight criteria $R_1, R_2, \ldots, R_8$ are obtained as

$\xi_1 = 0.328,\ \xi_2 = 0.220,\ \xi_3 = 0.156,\ \xi_4 = 0.105,\ \xi_5 = 0.072,\ \xi_6 = 0.049,\ \xi_7 = 0.041,\ \xi_8 = 0.029.$

Step 4. The decision matrices, expressed in linguistic variables is presented in Table 5. The p,q-QOF decision matrix (as per Table 2) with their level of confidence is presented in Table 6.

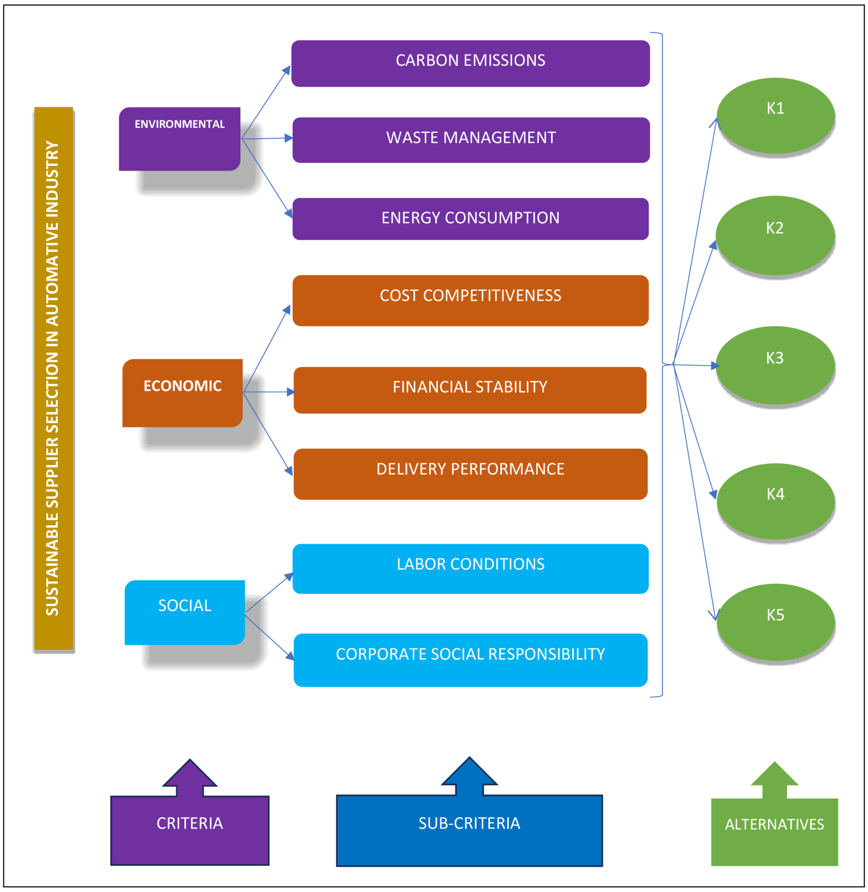

Fig. 2. Hierarchy for sustainable supplier selection

Table 3
Pairwise comparison matrix

	R1	R2	R3	R4	R5	R6	R7	R8
R1	1	3	2	4	5	6	6	7
R2	1/3	1	1/2	2	3	4	4	5
R3	1/2	2	1	3	3	5	5	6
R4	1/4	1/2	1/3	1	2	3	3	4
R5	1/5	1/3	1/3	1/2	1	2	2	3
R6	1/6	1/4	1/5	1/3	1/2	1	2	2
R7	1/6	1/4	1/5	1/3	1/2	1/2	1	2
R8	1/7	1/5	1/6	1/4	1/3	1/2	1/2	1

Table 4
Normalized pairwise comparison matrix

	R1	R2	R3	R4	R5	R6	R7	R8
R1	0.276	0.397	0.360	0.350	0.326	0.273	0.255	0.233
R2	0.092	0.132	0.090	0.175	0.196	0.182	0.170	0.167
R3	0.138	0.265	0.180	0.263	0.196	0.227	0.213	0.200
R4	0.069	0.066	0.060	0.088	0.130	0.136	0.128	0.133
R5	0.055	0.044	0.060	0.044	0.065	0.091	0.085	0.100
R6	0.046	0.033	0.036	0.029	0.033	0.045	0.085	0.067
R7	0.046	0.033	0.036	0.029	0.033	0.023	0.043	0.067
R8	0.039	0.026	0.030	0.022	0.022	0.023	0.021	0.033

Table 5
The decision matrices given in LVs

DEs	Alternatives	Sub criteria							
		$R1$	$R2$	$R3$	$R4$	$R5$	$R6$	$R7$	$R8$
D1	$K1$	B	M	VB	M	B	G	VG	B
	$K2$	M	VB	M	G	VG	B	G	G
	$K3$	G	G	VG	M	M	VB	M	B
	$K4$	VG	B	M	VG	G	G	M	VB
	$K5$	B	M	G	B	M	VG	G	M
D2	$K1$	G	G	B	M	M	M	M	VG
	$K2$	M	VB	M	G	VG	B	G	G
	$K3$	G	VG	M	M	VB	B	G	VG
	$K4$	B	M	G	VG	M	VG	B	G
	$K5$	VG	G	M	B	VB	G	M	M
D3	$K1$	B	M	M	G	VG	G	B	M
	$K2$	G	VG	G	G	VG	M	M	B
	$K3$	M	G	B	M	G	B	M	VG
	$K4$	M	B	VG	M	G	G	B	VG
	$K5$	VG	VG	M	G	B	M	VG	B

Table 6
Decision matrix using p,q-QOFNs with confidence levels

DEs	Alternatives	Sub criteria							
		$R1$	$R2$	$R3$	$R4$	$R5$	$R6$	$R7$	$R8$
D1	$K1$	$\langle(0.25, 0.75), 0.90\rangle$	$\langle(0.45, 0.55), 0.85\rangle$	$\langle(0.10, 0.98), 0.88\rangle$	$\langle(0.45, 0.55), 0.80\rangle$	$\langle(0.25, 0.75), 0.92\rangle$	$\langle(0.75, 0.25), 0.83\rangle$	$\langle(0.98, 0.10), 0.87\rangle$	$\langle(0.25, 0.75), 0.84\rangle$
	$K2$	$\langle(0.45, 0.55), 0.89\rangle$	$\langle(0.10, 0.98), 0.86\rangle$	$\langle(0.45, 0.55), 0.91\rangle$	$\langle(0.75, 0.25), 0.82\rangle$	$\langle(0.98, 0.10), 0.90\rangle$	$\langle(0.25, 0.75), 0.85\rangle$	$\langle(0.75, 0.25), 0.87\rangle$	$\langle(0.75, 0.25), 0.83\rangle$
	$K3$	$\langle(0.75, 0.25), 0.88\rangle$	$\langle(0.75, 0.25), 0.87\rangle$	$\langle(0.98, 0.10), 0.91\rangle$	$\langle(0.45, 0.55), 0.84\rangle$	$\langle(0.45, 0.55), 0.80\rangle$	$\langle(0.10, 0.98), 0.89\rangle$	$\langle(0.45, 0.55), 0.86\rangle$	$\langle(0.25, 0.75), 0.83\rangle$
	$K4$	$\langle(0.98, 0.10), 0.92\rangle$	$\langle(0.25, 0.75), 0.85\rangle$	$\langle(0.45, 0.55), 0.90\rangle$	$\langle(0.98, 0.10), 0.89\rangle$	$\langle(0.75, 0.25), 0.88\rangle$	$\langle(0.75, 0.25), 0.84\rangle$	$\langle(0.45, 0.55), 0.83\rangle$	$\langle(0.10, 0.98), 0.87\rangle$
	$K5$	$\langle(0.25, 0.75), 0.90\rangle$	$\langle(0.45, 0.55), 0.87\rangle$	$\langle(0.75, 0.25), 0.89\rangle$	$\langle(0.25, 0.75), 0.86\rangle$	$\langle(0.45, 0.55), 0.84\rangle$	$\langle(0.98, 0.10), 0.90\rangle$	$\langle(0.75, 0.25), 0.91\rangle$	$\langle(0.45, 0.55), 0.83\rangle$
D2	$K1$	$\langle(0.75, 0.25), 0.87\rangle$	$\langle(0.75, 0.25), 0.86\rangle$	$\langle(0.25, 0.75), 0.84\rangle$	$\langle(0.45, 0.55), 0.80\rangle$	$\langle(0.45, 0.55), 0.85\rangle$	$\langle(0.45, 0.55), 0.88\rangle$	$\langle(0.45, 0.55), 0.90\rangle$	$\langle(0.98, 0.10), 0.92\rangle$
	$K2$	$\langle(0.45, 0.55), 0.91\rangle$	$\langle(0.10, 0.98), 0.85\rangle$	$\langle(0.45, 0.55), 0.83\rangle$	$\langle(0.75, 0.25), 0.89\rangle$	$\langle(0.98, 0.10), 0.88\rangle$	$\langle(0.25, 0.75), 0.86\rangle$	$\langle(0.75, 0.25), 0.87\rangle$	$\langle(0.75, 0.25), 0.90\rangle$
	$K3$	$\langle(0.75, 0.25), 0.92\rangle$	$\langle(0.98, 0.10), 0.89\rangle$	$\langle(0.45, 0.55), 0.87\rangle$	$\langle(0.45, 0.55), 0.84\rangle$	$\langle(0.10, 0.98), 0.85\rangle$	$\langle(0.25, 0.75), 0.83\rangle$	$\langle(0.75, 0.25), 0.86\rangle$	$\langle(0.98, 0.10), 0.91\rangle$
	$K4$	$\langle(0.25, 0.75), 0.85\rangle$	$\langle(0.45, 0.55), 0.88\rangle$	$\langle(0.75, 0.25), 0.83\rangle$	$\langle(0.98, 0.10), 0.87\rangle$	$\langle(0.45, 0.55), 0.89\rangle$	$\langle(0.98, 0.10), 0.86\rangle$	$\langle(0.25, 0.75), 0.85\rangle$	$\langle(0.75, 0.25), 0.84\rangle$
	$K5$	$\langle(0.98, 0.10), 0.91\rangle$	$\langle(0.75, 0.25), 0.92\rangle$	$\langle(0.45, 0.55), 0.89\rangle$	$\langle(0.25, 0.75), 0.88\rangle$	$\langle(0.10, 0.98), 0.85\rangle$	$\langle(0.75, 0.25), 0.90\rangle$	$\langle(0.45, 0.55), 0.86\rangle$	$\langle(0.45, 0.55), 0.87\rangle$
D3	$K1$	$\langle(0.25, 0.75), 0.84\rangle$	$\langle(0.45, 0.55), 0.88\rangle$	$\langle(0.45, 0.55), 0.87\rangle$	$\langle(0.75, 0.25), 0.90\rangle$	$\langle(0.98, 0.10), 0.91\rangle$	$\langle(0.75, 0.25), 0.83\rangle$	$\langle(0.25, 0.75), 0.85\rangle$	$\langle(0.45, 0.55), 0.86\rangle$
	$K2$	$\langle(0.75, 0.25), 0.92\rangle$	$\langle(0.98, 0.10), 0.91\rangle$	$\langle(0.75, 0.25), 0.85\rangle$	$\langle(0.75, 0.25), 0.88\rangle$	$\langle(0.98, 0.10), 0.89\rangle$	$\langle(0.45, 0.55), 0.90\rangle$	$\langle(0.45, 0.55), 0.86\rangle$	$\langle(0.25, 0.75), 0.83\rangle$
	$K3$	$\langle(0.45, 0.55), 0.87\rangle$	$\langle(0.75, 0.25), 0.86\rangle$	$\langle(0.25, 0.75), 0.84\rangle$	$\langle(0.45, 0.55), 0.88\rangle$	$\langle(0.75, 0.25), 0.91\rangle$	$\langle(0.25, 0.75), 0.90\rangle$	$\langle(0.45, 0.55), 0.89\rangle$	$\langle(0.98, 0.10), 0.85\rangle$
	$K4$	$\langle(0.45, 0.55), 0.83\rangle$	$\langle(0.25, 0.75), 0.85\rangle$	$\langle(0.98, 0.10), 0.90\rangle$	$\langle(0.45, 0.55), 0.89\rangle$	$\langle(0.75, 0.25), 0.84\rangle$	$\langle(0.75, 0.25), 0.87\rangle$	$\langle(0.25, 0.75), 0.83\rangle$	$\langle(0.98, 0.10), 0.88\rangle$
	$K5$	$\langle(0.98, 0.10), 0.90\rangle$	$\langle(0.98, 0.10), 0.89\rangle$	$\langle(0.45, 0.55), 0.86\rangle$	$\langle(0.75, 0.25), 0.84\rangle$	$\langle(0.25, 0.75), 0.88\rangle$	$\langle(0.45, 0.55), 0.83\rangle$	$\langle(0.98, 0.10), 0.90\rangle$	$\langle(0.25, 0.75), 0.85\rangle$

Step 5. The individual decision matrices provided by the three decision experts (DEs) are aggregated using Eq. (7), where the parameters are set as $p = 4$, $q = 3$, and $\eta = 5$. The resulting aggregated decision matrix is presented in Table 7.

Table 7
Aggregated decision matrix

	R1	R2	R3	R4	R5	R6	R7	R8
K1	(0.717, 0.272)	(0.717, 0.272)	(0.42, 0.595)	(0.713, 0.274)	(0.974, 0.110)	(0.735, 0.26)	(0.976, 0.107)	(0.975, 0.108)
K2	(0.714, 0.274)	(0.974, 0.110)	(0.712, 0.275)	(0.746, 0.253)	(0.980, 0.101)	(0.42, 0.594)	(0.738, 0.258)	(0.738, 0.258)
K3	(0.739, 0.257)	(0.975, 0.109)	(0.976, 0.107)	(0.447, 0.555)	(0.713, 0.274)	(0.242, 0.769)	(0.717, 0.272)	(0.977, 0.104)
K4	(0.976, 0.107)	(0.424, 0.589)	(0.974, 0.11)	(0.978, 0.103)	(0.736, 0.259)	(0.975, 0.109)	(0.427, 0.584)	(0.974, 0.110)
K5	(0.978, 0.104)	(0.974, 0.110)	(0.724, 0.267)	(0.711, 0.275)	(0.427, 0.583)	(0.976, 0.107)	(0.974, 0.110)	(0.44, 0.565)

Step 6. Next, the combined values for each alternative with respect to each criterion are determined by applying Eq. (8). The aggregated results of each alternative are presented in Table 8.

Table 8
Ranking outcomes

Alternative	Aggregated value	Score value	Rank
$K1$	(0.807, 0.269)	0.7023	5
$K2$	(0.878, 0.214)	0.7922	4
$K3$	(0.905, 0.208)	0.8309	3
$K4$	(0.946, 0.178)	0.8976	2
$K5$	(0.946, 0.161)	0.8984	1

Step 7. Following this, the score values of the aggregated results are computed using Eq. (1), and the alternatives are ranked accordingly. The score values and final rankings are presented in Table 8.

Step 8. Based on Table 8, the ranking order of the five suppliers is determined as $K5 \succ K4 \succ K3 \succ K2 \succ K1$. Therefore, $K5$ is identified as the best supplier using the proposed method.

5.4 *Comparative Analysis*

This section provides a comparative analysis of the results produced by the proposed method and those derived from several existing techniques. For this purpose, different aggregation operator-based approaches are applied to the same numerical example (presented in Section 5.2), and the alternatives $K1$ through $K5$ are evaluated based on their score values $\Phi(K_i)$, which are then used to determine the final ranking. These methods include the confidence levels-based p, q-QOF weighted averaging/geometric (p,q-CQOFWA/p,q-CQOFWG) operators [41] (assuming $p = 4$, $q = 3$) and the confidence q-rung orthopair fuzzy weighted averaging/geometric (CFWA$_q$, CFWG$_q$) operators (assuming $p = q = 3$).

The comparative results are presented in Table 9 and Figure 3.

The key observations from Table 9 are summarized as follows:

- Proposed Method (CLp,q-QOFDWA): The proposed method produces the **highest score values** across all five alternatives, ranging from 0.7023 (K1) to 0.8984 (K5). This wide range indicates strong discriminatory power. The resulting ranking is: $K5 \succ K4 \succ K3 \succ K2 \succ K1$. This

Table 9
Comparison with the current approach of averaging aggregation method.

Methods	Score Values					Ranking
	$\Phi(K1)$	$\Phi(K2)$	$\Phi(K3)$	$\Phi(K4)$	$\Phi(K5)$	
p,q-CQOFWA [41]	0.5130	0.6277	0.6722	0.7218	0.7370	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
p,q-CQOFWG [41]	0.4306	0.5805	0.5841	0.5251	0.5050	$K3 \succ K2 \succ K4 \succ K5 \succ K1$
CFWA$_q$ [24]	0.5006	0.6156	0.6606	0.7056	0.7226	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
CFWG$_q$ [24]	0.4306	0.5805	0.5841	0.5251	0.5050	$K3 \succ K2 \succ K4 \succ K5 \succ K1$
CLp,q-QOFDWA (Proposed)	0.7023	0.7922	0.8309	0.8976	0.8984	$K5 \succ K4 \succ K3 \succ K2 \succ K1$

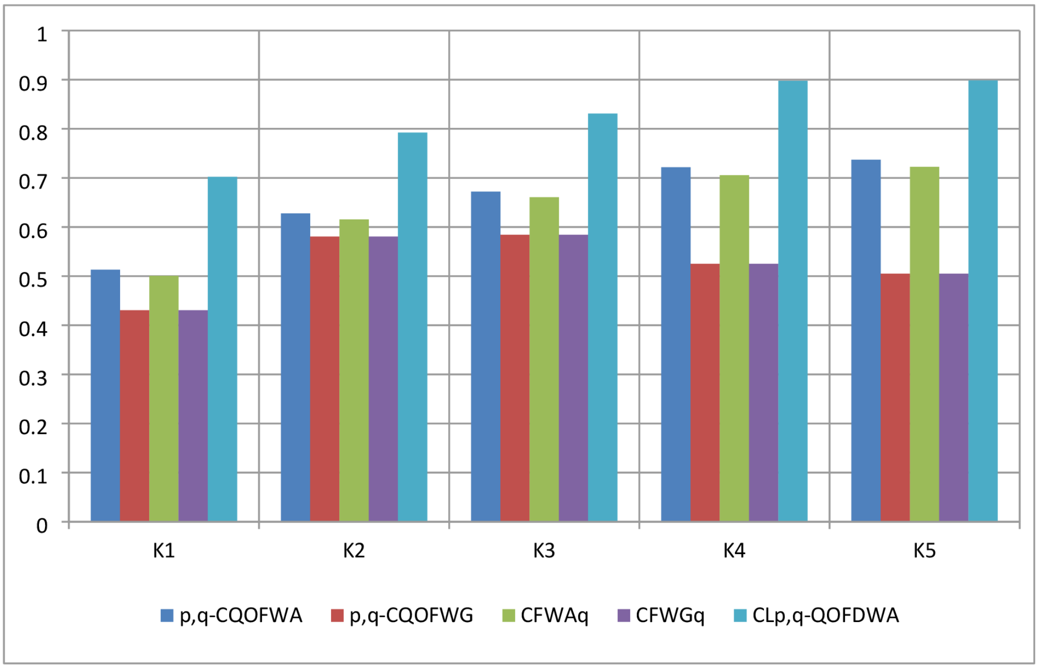

Fig. 3. Comparative analysis

ranking clearly identifies $K5$ as the most preferred alternative, closely followed by $K4$, and shows consistency and robustness in outcome.

- p,q-CQOFWA [41]: This method yields moderate score values (0.5130 to 0.7370), with the same ranking as the proposed method: $K5 \succ K4 \succ K3 \succ K2 \succ K1$. While the ranking matches, the score range is narrower, indicating less sensitivity in differentiating between alternatives.

- CFWA$_q$ [24]: This method also results in the same ranking: $K5 \succ K4 \succ K3 \succ K2 \succ K1$. However, the scores (0.5006 to 0.7226) are slightly lower and less spread than the proposed method, suggesting lower precision in ranking closeness among alternatives.

- p,q-CQOFWG [41] and CFWG$_q$ [24]: These geometric aggregation methods provide a distinctly different ranking: $K3 \succ K2 \succ K4 \succ K5 \succ K1$. In these cases, $K3$ is ranked highest and $K5$, which is top-ranked in most other methods, is pushed to fourth place. The score values (0.4306 to 0.5841) are also significantly compressed, reflecting weaker discriminative ability and potential inconsistency in outcome under the given decision context.

Therefore, the proposed CLp,q-QOFDWA operator based method demonstrates superior performance compared to existing methods by producing higher, more distinct score values and yielding a consistent and interpretable ranking order. This confirms its reliability and practical effectiveness for solving complex multi-criteria group decision-making problems in uncertain environments.

5.5 Sensitivity Analysis

The sensitivity analysis of the parameter η plays a crucial role in understanding the stability and robustness of the decision-making model. The parameter η is a crucial component in the Dombi aggregation operators, influencing the aggregation process. For solving the numerical problem, we initially set $\eta = 1$ arbitrarily. To conduct the sensitivity analysis, we systematically vary the value of η from 1 to 100, solving the numerical problem for each η and analyzing its effect on the ranking outcomes.

Table 10 presents the sensitivity analysis of the parameter η and its impact on the overall performance scores $\Phi(K_i)$ and rankings of the five alternatives K_1 to K_5. The parameter η plays a crucial role in influencing the aggregation behavior, and hence the final ranking of alternatives.

From Table 10, we observe that as the value of η increases from 1 to 100, the performance scores of all alternatives gradually increase. However, the rate of increase varies across the alternatives. Notably, the ranking of the alternatives remains relatively stable across a wide range of η, with some slight fluctuations in the top positions.

- At $\eta = 1$, the ranking is $K5 \succ K4 \succ K3 \succ K2 \succ K1$, indicating that $K5$ has the highest performance score, closely followed by $K4$.

- As η increases to 2, 3, and 4, $K4$ temporarily overtakes $K5$, becoming the top-ranked alternative. This suggests that for lower values of η, the model slightly favors $K4$.

- At $\eta = 5$, $K5$ regains the lead, although the performance scores of $K4$ and $K5$ remain very close, resulting in occasional switching between the first and second ranks.

- From $\eta = 6$ to approximately $\eta = 20$, the top position oscillates between $K4$ and $K5$, revealing sensitivity to small changes in η in this range.

- Beyond $\eta = 25$, the ranking stabilizes to $K5 \succ K4 \succ K3 \succ K2 \succ K1$, and this order remains consistent even as η increases up to 100.

Table 10
Sensitivity of the parameter η

η	$\Phi(K1)$	$\Phi(K2)$	$\Phi(K3)$	$\Phi(K4)$	$\Phi(K5)$	**Rank**
1	0.6101	0.7135	0.7606	0.8343	0.8355	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
2	0.6655	0.7617	0.8038	0.8751	0.8744	$K4 \succ K5 \succ K3 \succ K2 \succ K1$
3	0.6853	0.7782	0.8187	0.8888	0.8880	$K4 \succ K5 \succ K3 \succ K2 \succ K1$
4	0.6954	0.7852	0.8263	0.8941	0.8932	$K4 \succ K5 \succ K3 \succ K2 \succ K1$
5	0.7023	0.7922	0.8309	0.8976	0.8984	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
6	0.7058	0.7952	0.8340	0.9011	0.9001	$K4 \succ K5 \succ K3 \succ K2 \succ K1$
7	0.7093	0.7980	0.8341	0.9012	0.9002	$K4 \succ K5 \succ K3 \succ K2 \succ K1$
8	0.7117	0.7981	0.8356	0.9029	0.9002	$K4 \succ K5 \succ K3 \succ K2 \succ K1$
9	0.7140	0.8009	0.8371	0.9030	0.9036	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
10	0.7152	0.8010	0.838	0.9030	0.9036	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
9	0.7140	0.8009	0.8371	0.9030	0.9036	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
10	0.7152	0.8010	0.8387	0.9030	0.9036	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
11	0.7153	0.8024	0.8403	0.9047	0.9037	$K4 \succ K5 \succ K3 \succ K2 \succ K1$
12	0.7176	0.8038	0.8403	0.9048	0.9054	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
13	0.7177	0.8038	0.8403	0.9048	0.9054	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
14	0.7188	0.8053	0.8403	0.9065	0.9054	$K4 \succ K5 \succ K3 \succ K2 \succ K1$
15	0.7189	0.8053	0.8403	0.9065	0.9054	$K4 \succ K5 \succ K3 \succ K2 \succ K1$
20	0.7224	0.8082	0.8434	0.9082	0.9072	$K4 \succ K5 \succ K3 \succ K2 \succ K1$
25	0.7225	0.8082	0.8434	0.9083	0.9089	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
30	0.7236	0.8082	0.8435	0.9083	0.9089	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
35	0.7237	0.8082	0.8434	0.9083	0.9089	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
40	0.7237	0.8082	0.8450	0.9083	0.9089	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
45	0.7249	0.8097	0.8450	0.9083	0.9089	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
50	0.7249	0.8097	0.8465	0.9101	0.9086	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
55	0.7261	0.8111	0.8465	0.9101	0.9107	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
60	0.7261	0.8111	0.8465	0.9101	0.9107	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
65	0.7261	0.8111	0.8465	0.9101	0.9107	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
70	0.7272	0.8111	0.8465	0.9101	0.9107	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
75	0.7273	0.8111	0.8466	0.9101	0.9107	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
80	0.7273	0.8111	0.8466	0.9101	0.9107	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
90	0.7273	0.8111	0.8466	0.9101	0.9107	$K5 \succ K4 \succ K3 \succ K2 \succ K1$
100	0.7273	0.8112	0.8466	0.9101	0.9107	$K5 \succ K4 \succ K3 \succ K2 \succ K1$

It is also important to note that the relative ordering among the lower-ranked alternatives remains completely stable across all values of η: specifically, $K_3 \succ K_2 \succ K_1$ holds true for every tested value. This consistency reflects the robustness of the model in differentiating among the lower-performing options, regardless of the value of the sensitivity parameter. The only variations occur between the top two alternatives, $K4$ and $K5$, which switch positions depending on the value of η.

This analysis highlights that while η affects the magnitude of the performance scores, the ranking becomes robust for higher values of η (typically beyond $\eta = 25$). Hence, the model exhibits a degree of sensitivity to the parameter η in the lower range, especially in determining the top-ranked alternative. However, once η surpasses a certain threshold, the rankings become stable and reliable, indicating robustness of the decision-making process under higher values of the parameter.

To visually illustrate the trend of sensitivity analysis, Figure 4 presents a graphical representation of the variation in scores $\Phi(Ki)$ for different values of η.

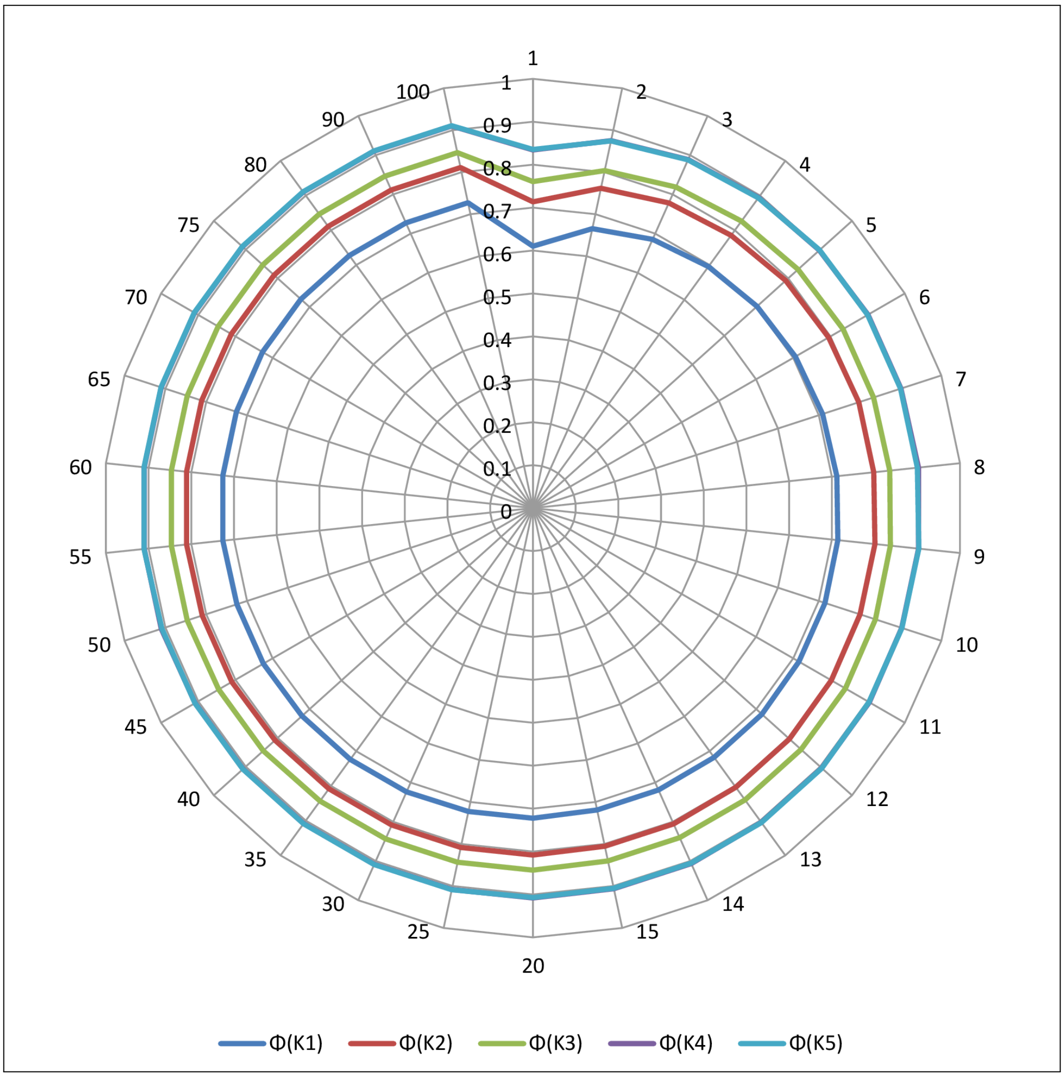

Fig. 4. Sensitivity analysis of the parameter η

6. Conclusion

This study proposes CL-based Dombi weighted averaging and geometric aggregation operators for p,q-QOFSs to support decision-making processes. Several important properties of these operators have been well established. By integrating experts' confidence levels into the evaluation process, these operators more effectively model real-world scenarios, offering a more realistic and reliable representation within the p,q-QOF framework. These operators are incorporated into a MCGDM model to demonstrate their effectiveness in capturing real-world scenarios, particularly when expert confidence levels are considered during the evaluation process. In this model, the criteria weights are computed by using AHP method. A numerical example focused on selecting a sustainable supplier for the automotive industry has been presented to showcase the applicability and advantages of the proposed method, with a comparative analysis against existing approaches. In this problem, the decision-making process involved input from three decision experts, who evaluated three alternative suppliers based on three major sustainable criteria, further broken down into eight detailed sub-criteria. The results of the analysis demonstrated that supplier $K5$ was identified as the most suitable and optimal supplier, offering the best performance with respect to the defined sustainability criteria. Furthermore, a detailed sensitivity analysis was performed to assess the stability and reliability of the results. The findings confirmed that the proposed model is stable under variations in parameter settings and provides consistent outcomes, thereby enhancing its credibility for practical applications.

This study offers important insights but has certain limitations. Specifically, the number of experts and alternatives considered in the numerical example is limited, primarily to maintain clarity and manageability in the illustration. Nevertheless, the proposed model is designed to be flexible and scalable, making it applicable to decision-making problems involving any number of alternatives and criteria. Future research could explore its effectiveness in more complex and large-scale scenarios. Again, the current study focuses primarily on the development of the proposed aggregation model; future work will incorporate comparative analyzes using methods such as EDAS and PROMETHEE to enhance validation and robustness.

In the future, additional CL-based AOs can be developed for advanced extensions of fuzzy sets by employing various TNs and TCNs. Future research can be extended to address a wide range of decision-making challenges, feature selection problem, medical diagnosis and problems of recruitment and selection, etc. Moreover, the proposed approach has the potential to be utilized within different uncertainty environments, such as neutrosophic fuzzy sets, hesitant fuzzy sets, and beyond.

Acknowledgement

This research was not funded by any grant.

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] Kolour, H. R., Momayezi, V., & Momayezi, F. (2025). Enhancing supplier selection in public manufacturing: A hybrid multi-criteria decision-making approach. *Spectrum of Decision Making and Applications*, *3*(1), 1–20. https://doi.org/10.31181/sdmap31202629

[2] Wang, Y., Yang, H., & Han, X. (2024). Study on the method of selecting sustainable food suppliers considering interactive factors. *Journal of Operations Intelligence*, *2*(1), 202–218. https://doi.org/10.31181/jopi21202420

[3] Mandal, U., & Seikh, M. R. (2023a). An integrated weighted distance-based approximation method for interval-valued spherical fuzzy magdm. In C. Jana, M. Pal, G. Muhiuddin, & P. Liu (Eds.), *Fuzzy optimization, decision-making and operations research*. Springer, Cham. https://doi.org/10.1007/978-3-031-35668-1_24

[4] Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, *8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X
[5] Seikh, M. R., & Mandal, U. (2022). Multiple attribute group decision making based on quasirung orthopair fuzzy sets: Application to electric vehicle charging station site selection problem. *Engineering Applications of Artificial Intelligence*, *115*, 105299. https://doi.org/10.1016/j.engappai.2022.105299
[6] Rahim, M., Abosuliman, S. S., Alroobaea, R., Shah, K., & Abdeljawad, T. (2024). Cosine similarity and distance measures for p, q quasirung orthopair fuzzy sets: Applications in investment decision-making. *Heliyon*, *10*(11). https://doi.org/10.1016/j.heliyon.2024.e32107
[7] Rahim, M., Akhtar, Y., Yang, M. S., Ali, H. E., & Elhag, A. A. (2024). Improved copras method with unknown weights under p, q-quasirung orthopair fuzzy environment: Application to green supplier selection. *IEEE Access*, *12*, 69783–69795. https://doi.org/10.1109/ACCESS.2024.3400016
[8] Alballa, T., Rahim, M., Aloraini, N. M., & Khalifa, H. A. E. W. (2024). An extension of the codas method using p,q-quasirung orthopair fuzzy information: Application in location selection for retail store. *International Journal of Fuzzy Systems*, 1–17. https://doi.org/10.1007/s40815-024-01870-5
[9] Rahim, M., Tag Eldin, E. M., Khan, S., Ghamry, N. A., Alanzi, A. M., & Khalifa, H. A. E. W. (2024). Multi-criteria group decision-making based on dombi aggregation operators under p,q-quasirung orthopair fuzzy sets. *Journal of Intelligent & Fuzzy Systems*, *46*(1), 53–74. https://doi.org/10.3233/JIFS-233327
[10] Rahim, M., Ahmad, S., Younis, B. A., Egami, R. H., & Ahmed, M. M. (2025). Multiple attribute group decision making based on p, q-quasirung orthopair bonferroni mean operators and their applications. *Evolving Systems*, *16*(1), 9. https://doi.org/10.1007/s12530-024-09638-w
[11] Ali, J., & Naeem, M. (2023). Analysis and application of p, q-quasirung orthopair fuzzy aczel–alsina aggregation operators in multiple criteria decision-making. *IEEE Access*, *11*, 49081–49101. https://doi.org/10.1109/ACCESS.2023.3274494
[12] Arya, P., & Pal, A. K. (2025). Mcdm model using jaccard and cosine similarity-driven aggregation operators in (n, m)-rung orthopair fuzzy environment: A case study on government medical facilities in indian states. *International Journal of Information Technology*, *17*(1), 225–236. https://doi.org/10.1007/s41870-024-02233-x
[13] Dombi, J. (1982). A general class of fuzzy operators, the de morgan class of fuzzy operators and fuzziness measure included by fuzzy operators. *Fuzzy Sets and Systems*, *8*(2), 149–163. https://doi.org/10.1016/0165-0114(82)90005-7
[14] Seikh, M. R., & Mandal, U. (2021). Intuitionistic fuzzy dombi aggregation operators and their application to multiple attribute decision-making. *Granular Computing*, *6*, 473–488. https://doi.org/10.1007/s41066-019-00209-y
[15] Alolaiyan, H., Kalsoom, U., Shuaib, U., Razaq, A., Baidar, A. W., & Xin, Q. (2024). Precision measurement for effective pollution mitigation by evaluating air quality monitoring systems in linguistic pythagorean fuzzy dombi environment. *Scientific Reports*, *14*(1), 31944. https://doi.org/10.1038/s41598-024-83478-1
[16] Saha, A., Dabic-Miletic, S., Senapati, T., Simic, V., Pamucar, D., Ala, A., & Arya, L. (2024). Fermatean fuzzy dombi generalized maclaurin symmetric mean operators for prioritizing bulk material handling technologies. *Cognitive Computation*, *16*, 3096–3121. https://doi.org/10.1007/s12559-024-10323-y
[17] Seikh, M. R., & Mandal, U. (2023). Interval-valued fermatean fuzzy dombi aggregation operators and swara based promethee ii method to bio-medical waste management. *Expert Systems with Applications*, *226*, 120082. https://doi.org/10.1016/j.eswa.2023.120082

[18] Senapati, T., Chen, G., Ullah, I., Khan, M. S. A., & Hussain, F. (2024). A novel approach towards multiattribute decision making using q-rung orthopair fuzzy dombi–archimedean aggregation operators. *Heliyon*, *10*(6). https://doi.org/10.1016/j.heliyon.2024.e27969
[19] Mandal, U., & Seikh, M. R. (2023b). Interval-valued spherical fuzzy mabac method based on dombi aggregation operators with unknown attribute weights to select plastic waste management process. *Applied Soft Computing*, *145*, 110516. https://doi.org/10.1016/j.asoc.2023.110516
[20] Kavitha, S., Janani, K., Mohanrasu, S. S., Satheeshkumar, J., Amudha, T., & Rakkiyappan, R. (2024). Ensemble feature selection using q-rung orthopair hesitant fuzzy hamacher, einstein and dombi aggregation operators. *Applied Soft Computing*, *161*, 111752. https://doi.org/10.1016/j.asoc.2024.111752
[21] Seikh, M. R., & Chatterjee, P. (2025). Sustainable strategies for electric vehicle adoption: A confidence level-based interval-valued spherical fuzzy merec-vikor approach. *Information Sciences*, *699*, 121814. https://doi.org/10.1016/j.ins.2024.121814
[22] Seikh, M. R., & Chatterjee, P. (2024). Evaluation and selection of e-learning websites using intuitionistic fuzzy confidence level based dombi aggregation operators with unknown weight information. *Applied Soft Computing*, *163*, 111850. https://doi.org/10.1016/j.asoc.2024.111850
[23] Chatterjee, P., & Seikh, M. R. (2024). Evaluating municipal solid waste management with a confidence level-based decision-making approach in q-rung orthopair picture fuzzy environment. *Journal of Industrial Information Integration*, *42*, 100708. https://doi.org/10.1016/j.jii.2024.100708
[24] Joshi, B. P., & Gegov, A. (2020). Confidence levels q-rung orthopair fuzzy aggregation operators and its applications to mcdm problems. *International Journal of Intelligent Systems*, *35*(1), 125–149. https://doi.org/10.1002/int.22203
[25] Mahmood, T., Ali, Z., & Yang, M. (2023). Confidence level aggregation operators based on intuitionistic fuzzy rough sets with application in medical diagnosis. *IEEE Access*, *11*, 8674–8688. https://doi.org/10.1109/ACCESS.2023.3236410
[26] Lin, H., Ullah, I., Ali, A., & Abbas, S. (2023). Analysis of cost and profit using aggregation operator on spherical fuzzy sets with confidence level. *Journal of Intelligent and Fuzzy Systems*, *45*(1), 675–686. https://doi.org/10.3233/JIFS-220102
[27] Punetha, T., & Komal. (2024). Confidence picture fuzzy hybrid aggregation operators and its application in mcgdm. *OPSEARCH*, 1–37. https://doi.org/10.1007/s12597-023-00720-6
[28] Qiyaas, M., Khan, N., Khan, S., & Khan, F. (2024). Confidence levels bipolar complex fuzzy aggregation operators and their application in decision making problem. *IEEE Access*, *12*, 6204–6214. https://doi.org/10.1109/ACCESS.2023.3347043
[29] Sivadas, A., John, S. J., & Athira, T. (2024). (p, q)-fuzzy aggregation operators and their applications to decision-making. *The Journal of Analysis*, 1–30. https://doi.org/10.1007/s41478-023-00693-1
[30] Saaty, T. L. (1980). The analytic hierarchy process (ahp). *The Journal of the Operational Research Society*, *41*(11), 1073–1076.
[31] Tronnebati, I., Jawab, F., Frichi, Y., & Arif, J. (2024). Green supplier selection using fuzzy ahp, fuzzy tosis, and fuzzy waspas: A case study of the moroccan automotive industry. *Sustainability*, *16*(11), 4580. https://doi.org/10.3390/su16114580
[32] Kara, K., Acar, A. Z., Polat, M., Önden, İ., & Yalçın, G. C. (2024). Developing a hybrid methodology for green-based supplier selection: Application in the automotive industry. *Expert Systems with Applications*, *249*, 123668. https://doi.org/10.1016/j.eswa.2024.123668
[33] Bas, S. A. (2024). A hybrid approach based on consensus decision making for green supplier selection in automotive industry. *Sustainability*, *16*(7), 3096. https://doi.org/10.3390/su16073096

[34] Gergin, R. E., Peker, İ., & Kısa, A. C. G. (2022). Supplier selection by integrated ifdematel-iftopsis method: A case study of automotive supply industry. *Decision Making: Applications in Management and Engineering*, *5*(1), 169–193. https://doi.org/10.31181/dmame211221075g

[35] Bah, M. K., & Tulkinov, S. (2022). Evaluation of automotive parts suppliers through ordinal priority approach and topsis. *Management Science and Business Decisions*, *2*(1), 5–17. https://doi.org/10.52812/msbd.37

[36] Dweiri, F., Kumar, S., Khan, S. A., & Jain, V. (2016). Designing an integrated ahp based decision support system for supplier selection in automotive industry. *Expert Systems with Applications*, *62*, 273–283. https://doi.org/10.1016/j.eswa.2016.06.030

[37] Dang, T. T., Nguyen, N. A. T., Nguyen, V. T. T., & Dang, L. T. H. (2022). A two-stage multi-criteria supplier selection model for sustainable automotive supply chain under uncertainty. *Axioms*, *11*(5), 228. https://doi.org/10.3390/axioms11050228

[38] Zimmer, K., Fröhling, M., & Schultmann, F. (2016). Sustainable supplier management–a review of models supporting sustainable supplier selection, monitoring and development. *International journal of production research*, *54*(5), 1412–1442. https://doi.org/10.1080/00207543.2015.1079340

[39] Masudin, I., Habibah, I. Z., Wardana, R. W., Restuputri, D. P., & Shariff, S. S. R. (2024). Enhancing supplier selection for sustainable raw materials: A comprehensive analysis using analytical network process (anp) and topsis methods. *Logistics*, *8*(3), 74. https://doi.org/10.3390/logistics8030074

[40] Amindoust, A., Ahmed, S., Saghafinia, A., & Bahreininejad, A. (2012). Sustainable supplier selection: A ranking model based on fuzzy inference system. *Applied soft computing*, *12*(6), 1668–1677. https://doi.org/10.1016/j.asoc.2012.01.023

[41] Rahim, M., Shah, K., Abdeljawad, T., Aphane, M., Alburaikan, A., & Khalifa, H. A. E.-W. (2023). Confidence levels-based p, q-quasirung orthopair fuzzy operators and its applications to criteria group decision making problems. *IEEE Access*, *11*, 109983–109996. https://doi.org/10.1109/ACCESS.2023.3321876